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Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

William Blastall Wilkinson, Lieut. R.N.R., H.M.S. *Encounter*, Sydney, New South Wales, Australia,

was balloted for and duly elected a Fellow of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

H. D. Warburg, Lieut.-Commander R.N., Hydrographic Department, Admiralty, Whitehall, S.W. (proposed by Canon A. M. Deane).

Seventy presents were announced as having been received since the last ordinary meeting, including, amongst others:

Tables annuelles de Constantes et Données Numériques (Données numériques de Spectroscopie), presented by the Editors; Lick Observatory Publications, vol. xi. (Photographs of the Milky Way and Comets, made during the years 1892–95, by E. E. Barnard), presented by the Director.

The Dynamics of a Globular Stellar System. Second Paper. By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. In the first paper * I have given my reasons for believing that the stars move in undisturbed paths under the general attraction of the whole stellar system without experiencing any appreciable deflection by the chance encounters or approaches of other stars. This conclusion has been strengthened by J. H. Jeans's calculation of the probable amount of such deflections.† The "free-path" of a star is accordingly taken to be infinitely great, and this provides a basis for investigating the dynamics of stellar motions. In the first paper, as an introduction to the subject, some examples of simple states of steady motion were worked out; but I did not then succeed in my main object of finding a steady system in which the distribution of stellar velocities would approximate to that which we actually observe.

In the present paper a solution of this problem is found. Schwarzschild's ellipsoidal hypothesis is taken to represent the observed distribution of velocities, and a steady state of motion is found in which Schwarzschild's law is satisfied exactly. Thus the phenomenon of two star-streams is satisfactorily represented in this model stellar system, following the well-known suggestion of H. H. Turner.‡ Moreover, it is found that there need be *no congestion of stars at the centre of the system*, the increase of density towards the centre being fairly small.

Before proceeding to the mathematical discussion, we may look at the problem from an elementary point of view. We know from observation the distribution of velocities of the stars in our own neighbourhood. Now these stars have travelled in from other parts of the system; and, therefore, if we trace back their paths and their changes of velocity, we shall learn something of the distribution of stellar motions and of the number of stars in these other parts. We can thus, to a certain extent, build up the complete system from our knowledge of the part in our neighbourhood.§ The practical difficulty is that we cannot trace the paths until we know the law of force in the system; this law of force depends on the density or number of stars at different distances from the centre; but the density is one of the things we are trying to learn by tracing back the paths. We are thus necessarily working in a circle, in which the conclusion must not contradict the premises. To solve the problem means to find a law of

* *Monthly Notices*, vol. lxxiv. p. 5. † *Ibid.*, p. 109.

‡ *Ibid.*, vol. lxxii. p. 387.

§ This relation between the motions in different parts of the system is equivalent to an equation of continuity. In the particular case of globular symmetry the equation of continuity can be integrated. In the following analysis I have passed at once to the integrated form, so that the connection with the well-known differential equation of continuity is not immediately obvious.

density, etc., for which there is no such contradiction; and that is the property (and value) of the solution found below. Also, since there must be stars which never pass through the region near our sun, it must be possible to vary the solution whilst still satisfying the conditions. This is provided for by the arbitrary constants, which appear on integrating the differential equation.

We shall find in the mathematical solution that the hypothesis of an ellipsoidal distribution of velocities, together with the assumption of globular symmetry, suffices to determine *uniquely* (save for arbitrary constants) the density and state of motion throughout the whole system. The uniqueness of the solution is, however, not a fact on which we need lay stress from the astronomical standpoint. It is doubtful if we can, in practice, deduce the nature of the whole system from our knowledge of a small part. My object has been much more limited; it is to find some self-consistent dynamical model in which the curious law of velocities deduced from observation is properly represented, which may, perhaps, be a useful accessory in practical researches.

2. The limitations are the same as in the previous paper, and are as follows:—

(*a*) The stars are supposed to move under the central attraction of the whole system, undisturbed by their immediate neighbours.

(*b*) A search is made for distributions of velocity which yield a steady state for the whole system.

(*c*) The system is taken to be globular (*i.e.* to have spherical symmetry); the density is not necessarily uniform.

To these we now add:—

(*d*) The distribution of stellar velocities everywhere follows Schwarzschild's ellipsoidal law. The prolateness of the velocity-ellipsoid may vary in different parts of the system; near the sun it must have its observed value.

Let r be the distance of a star from the centre of the system.

u its radial velocity.

v its transverse velocity.

ϕ the gravitational potential at r.

σ the density at r.

Let the suffix o refer to the apcentron of the star's orbit, so that

r_0 is its distance at apcentron,

v_0 its velocity when at apcentron,

ϕ_0 the potential at r_0.

u_0 is, of course, zero.

Let γ_0 be the maximum possible value of v_0 corresponding to a given r_0. If the transverse velocity is greater than that required to maintain the star in a circular orbit, the star will begin to recede from the centre and consequently cannot be at apcentron. γ_0 is thus the velocity for a circular orbit, and we have

$$-\left(\frac{\partial\phi}{\partial r}\right)_0 = \frac{\gamma_0^2}{r_0}.$$

Hence
$$\gamma_0^2 = -r_0 \frac{\partial \phi_0}{\partial r_0} \quad . \quad . \quad . \quad . \quad (1)$$

The equations of energy and angular momentum give

$$\tfrac{1}{2}(u^2+v^2) - \phi = \tfrac{1}{2}v_0^2 - \phi_0 \quad . \quad . \quad . \quad (2)$$
$$rv = r_0 v_0 \quad . \quad . \quad . \quad . \quad (3)$$

In a time interval δt a certain number of stars will arrive at apcentron. Consider those with apcentron distance between r_0 and $r_0 + dr_0$, and with velocity at apcentron between v_0 and $v_0 + dv_0$. The number of these will be some function of r_0 and v_0, which we shall call *
$$f(r_0, v_0) dr_0 dv_0 \,.\, \delta t.$$

During the next circuit any one of these stars will be within the spherical shell r to $r+dr$ for the time $\frac{2dr}{u}$. The factor 2 allows for going and returning. The volume of the shell is $4\pi r^2 dr$. Hence these stars will supply a density (number of stars per unit volume) in the shell equal to

$$\frac{1}{4\pi r^2 dr} \,.\, f(r_0, v_0) dr_0 dv_0 \,.\, \delta t \quad \text{for a time } \frac{2dr}{u},$$

which is equivalent to a density

$$\frac{1}{2\pi r^2 u} \,.\, f(r_0, v_0) dr_0 dv_0 \quad \text{for a time } \delta t,$$

so far as the time average is concerned.

We thus find that the stars of character $dr_0 dv_0$ make a contribution $d^2\sigma$ to the density at r, given by

$$d^2\sigma = \frac{f(r_0, v_0) dr_0 dv_0}{2\pi r^2 u} \quad . \quad . \quad . \quad . \quad (4)$$

The whole density at r can then be obtained by a double integration over all values of r_0 and v_0.

3. We now proceed to change the variables from r_0, v_0 (which may be regarded as the elements of the stellar orbit) to u, v, the actual velocity-components of the star at r.†

From equations (2) and (3) we have

$$u^2 = 2(\phi - \phi_0) - v_0^2\left(\frac{r_0^2}{r^2} - 1\right) \quad . \quad . \quad . \quad (5)$$

* In the former paper the form $a(r_0) \,.\, f(v_0) dr_0 dv_0 \delta t$ was assumed ; but it was stated that $f(v_0)$ might be a function of r_0 as well as v_0.

† The advantage and practicability of this change of variables was suggested to me by reading a hitherto unpublished paper by Mr. H. Jeffreys on the distribution of meteoric matter in the solar system.

Hence
$$u\frac{\partial u}{\partial r_0} = -\frac{\partial \phi_0}{\partial r_0} - \frac{v_0^2 r_0}{r^2}$$
$$u\frac{\partial u}{\partial v_0} = -v_0\left(\frac{r_0^2}{r^2} - 1\right).$$

Also from (3)
$$r\frac{\partial v}{\partial r_0} = v_0 \qquad r\frac{\partial v}{\partial v_0} = r_0.$$

Hence
$$ru\frac{\partial(u, v)}{\partial(r_0, v_0)} = \begin{vmatrix} -\frac{\partial \phi_0}{\partial r_0} - \frac{v_0^2 r_0}{r^2}, & -v_0\left(\frac{r_0^2}{r^2} - 1\right) \\ v_0 \quad , & r_0 \end{vmatrix}$$
$$= -r_0\frac{\partial \phi_0}{\partial r_0} - v_0^2$$
$$= \gamma_0^2 - v_0^2 \text{ by equation (1).}$$

Accordingly
$$dudv = \frac{\partial(u, v)}{\partial(r_0, v_0)} dr_0 dv_0$$
$$= \frac{\gamma_0^2 - v_0^2}{ru} dr_0 dv_0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence
$$d^2\sigma = \frac{f(r_0, v_0) dudv}{2\pi r(\gamma_0^2 - v_0^2)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The element of density $d^2\sigma$ corresponding to a velocity range $dudv$, is the number of stars per unit volume having a velocity within that range. By hypothesis this is to conform to Schwarzschild's law, viz.
$$d^2\sigma = \mathrm{C}ve^{-h_1^2u^2 - h_2^2v^2} dudv \quad . \quad . \quad . \quad (8)$$

Here C, h_1, h_2 may be functions of r. The factor v occurs because v is a two-dimensional component, whereas u is a one-dimensional component of velocity. For convenience we shall write (8), with a different notation of coefficients, thus
$$d^2\sigma = \mathrm{C}ve^{-h^2(u^2+v^2)} \cdot e^{-k^2r^2v^2} dudv \quad . \quad . \quad (9)$$
where C, h, k may be functions of r.

Eliminating u and v by (2) and (3), we have
$$d^2\sigma = \frac{\mathrm{C}}{r} r_0 v_0 e^{-h^2v_0^2 + 2h^2\phi_0 - 2h^2\phi} \cdot e^{-k^2r_0^2v_0^2} dudv \quad . \quad (10)$$

Hence equating (10) and (7),
$$f(r_0, v_0) = 2\pi\mathrm{C}e^{-2h^2\phi} \cdot r_0v_0(\gamma_0^2 - v_0^2)e^{2h^2\phi_0} \cdot e^{-(h^2+k^2r_0^2)v_0^2} \quad (11)$$

Since this function must not involve r, we must have
$$2\pi\mathrm{C}e^{-2h^2\phi} = \text{constant} = \mathrm{B} \text{ say,}$$

and h and k must be *constants.**

* ϕ_0 and γ_0 are both functions of r_0. The foregoing argument is, in brief, that a certain function of the unsuffixed letters only (given as regards u and v, but undetermined as regards r) must transform into a function of the suffixed letters only.

Hence $$f(r_0, v_0) = Br_0v_0(\gamma_0{}^2 - v_0{}^2)e^{2h^2\phi_0} \cdot e^{-(h^2+k^2r_0{}^2)v_0{}^2} \quad . \quad (12)$$

and $$d^2\sigma = \frac{Be^{2h^2\phi}}{2\pi} \cdot ve^{-h^2u^2-(h^2+k^2r^2)v^2}dudv \quad . \quad . \quad (13)$$

4. Schwarzschild's ellipsoidal law provides no upper limit to the values of the velocities u, v of a star; but the frequency of high velocities diminishes so rapidly that for practical purposes it may be considered zero after a certain limit. In the mathematical solution, however, it makes some difference whether the expectation of a high velocity (say above 1000 km. per sec.) is strictly zero or only infinitesimal. In the former case we can set a boundary to the universe, only making it sufficiently extensive to provide a drop of potential great enough to give the maximum velocity of 1000 km. per sec. In the latter case, however infinitesimal may be the chance of a star exceeding that velocity, we must provide a drop of potential sufficient for the purpose. This can only be done by adding large numbers of stars in the outer parts of space, so that the potential continues to decrease outwards to the value $-\infty$ when r_0 is infinite. It does not matter in the least how few stars avail themselves of the large potential difference to get up a high velocity; if we admit the slightest expectation of such a velocity, the mathematical solution must provide the elaborate mechanism required to impart it.

We must therefore be prepared to find in the mathematical solution that ϕ_0 tends to $-\infty$, as r_0 tends to ∞. This involves an infinite number of stars in the outer parts of the system. But their function is purely passive; they are there to provide a force which is not needed in the practical problem. After obtaining the solution we are at liberty to disregard them, and finish off the system in some finite way.

5. Performing a double integration on (13) between the limits $+\infty$ and $-\infty$ for u, and 0 and ∞ for v, we obtain the density

$$\sigma = \frac{Be^{2h^2\phi}}{2\pi} \cdot \frac{1}{2(h^2+k^2r^2)} \cdot \frac{\sqrt{\pi}}{h} \cdot \quad . \quad . \quad (14)$$

Now $$\nabla^2\phi = -4\pi\sigma \text{ (Poisson's Equation),}$$

or since for globular symmetry

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr},$$

we have $$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \frac{B\sqrt{\pi}}{h} \cdot \frac{e^{2h^2\phi}}{h^2+k^2r^2} = 0 \quad . \quad . \quad (15)$$

By a suitable choice of units we can put $h = 1$, $k = 1$; and, since the zero of ϕ is arbitrary, the factor $\frac{B\sqrt{\pi}}{h}$ can be absorbed; so that the equation can be reduced to the form

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \frac{e^{2\phi}}{1+r^2} = 0 \quad . \quad . \quad . \quad (16)$$

Alternative forms are found by putting $u = \frac{1}{r}$, $\phi = -\log z$; these are

$$\frac{d^2\phi}{du^2} + \frac{e^{2\phi}}{u^2(1+u^2)} = 0 \quad . \quad . \quad . \quad . \quad (17)$$

and

$$z\frac{d^2z}{du^2} - \left(\frac{dz}{du}\right)^2 = \frac{1}{u^2(1+u^2)} \quad . \quad . \quad . \quad (18)$$

In the same units (14) becomes

$$4\pi\sigma = \frac{e^{2\phi}}{1+r^2} = \frac{1}{z^2(1+r^2)} \quad . \quad . \quad . \quad (19)$$

We have thus a differential equation for the potential and density in a globular system satisfying Schwarzschild's law.

6. The differential equation is closely analogous to an equation occurring in the investigation of gravitating spheres of gas, which has been carefully studied. The latter equation is known to be insoluble, and presumably this equation is insoluble likewise. We can, however, learn the chief properties of the distribution, and make numerical calculations for any particular case.

When u is large the equation is approximately

$$\frac{d^2\phi}{du^2} + \frac{e^{2\phi}}{u^4} = 0,$$

or more conveniently, $$z\frac{d^2z}{du^2} - \left(\frac{dz}{du}\right)^2 = \frac{1}{u^4},$$

and $\sigma \propto \frac{1}{z^2}$.

This is the law of potential in a gravitating sphere of gas at constant temperature, so that at small distances from the centre the density in the stellar system follows the same law as in an isothermal mass of gas.*

We may divide the solutions into three cases according as $\left(\frac{dz}{du}\right)^2$ is of order ultimately greater than, equal to, or less than $\frac{1}{u^4}$, when u becomes infinite.

Case (1). $\frac{dz}{du}$ is infinitely greater than u^{-2}, for $u = \infty$.

We may then neglect u^{-4} compared with $\left(\frac{dz}{du}\right)^2$, and hence

$$\left(\frac{dz}{du}\right)^2 = z\frac{d^2z}{du^2}.$$

* A comparison may be made with the work of v. Zeipel and Plummer on the distribution of stars in globular clusters. For the clusters the star-density approximates to an *adiabatic* mass of gas (*cf. Monthly Notices*, vol. lxxi. p. 460).

The complete solution of this is $z = Ae^{ku}$, where k is positive owing to the condition $\frac{dz}{du} > u^{-2}$. The density at the centre vanishes like e^{-2ku}.

It seems doubtful whether this solution can be continued outwards to the regions where u^{-4} becomes appreciable. In any case we shall not trouble to investigate further a stellar system with a hollow centre!

Case (2). $\frac{dz}{du}$ is equal to, or a finite multiple of, u^{-2}, for $u = \infty$.

This case has one solution, $z = u^{-1} = r$. In this case σ becomes infinite at the centre like r^{-2}. There is a rapid increase of density towards the centre, and the congestion of stars there is very considerable; but the number of stars in any finite region is not infinite, so that the solution can quite well correspond to an actual system.

The corresponding solution, valid for finite values of r, can be obtained in the form of a series, viz.

$$z = r - \frac{1}{8}r^3 + \frac{61}{1408}r^5 - \frac{2633}{123904}r^7 + \ \ldots$$

The series, however, diverges unless r is small.* In order to extend the solution to the outer parts of the system, we should have to proceed by quadrature step by step from the differential equation. This latter method has been applied in the analogous problem of the sphere of isothermal gas.

Case (3). $\frac{dz}{du}$ is infinitely smaller than u^{-2}, for $u = \infty$.

Here $\left(\frac{dz}{du}\right)^2$ can be neglected in comparison with u^{-4}, and the equation can conveniently be written (by adding the negligible term $2(dz/du)^2$)

$$z\frac{d^2z}{du^2} + \left(\frac{dz}{du}\right)^2 = \frac{1}{u^4}.$$

The complete solution is

$$z^2 = A + Bu + \frac{1}{3u^2} \qquad (u = \infty)$$

We see that B must be zero, and A cannot be zero; otherwise the solution comes under one of the preceding cases.

Hence $z^2 = A + \frac{1}{3}r^2$ near the centre.

This case is of great interest because the density is finite and the rate of increase towards the centre is slow. Evidently this is

* Taking the ratio of the axes of the velocity-ellipsoid near the sun to be 2 : 1 : 1, the distance of the sun from the centre is in the present units $r = \sqrt{3}$, much beyond the range of convergence.

the case in which may be found the most satisfactory approximation to our own stellar system.

7. To develop a solution of the third type of the equation

$$z\frac{d^2z}{du^2} - \left(\frac{dz}{du}\right)^2 = \frac{1}{u^2(1+u^2)}$$

we set $z^2 = A + \frac{1}{3}u^{-2} + a_4u^{-4} + a_6u^{-6} + a_8u^{-8} + a_{10}u^{-10} + \ldots.$

Then

$$-2z\frac{dz}{du} = \tfrac{2}{3}u^{-3} + 4a_4u^{-5} + 6a_6u^{-7} + 8a_8u^{-9} + 10a_{10}u^{-11} + \ldots.$$

$$2\left\{z\frac{d^2z}{du^2} + \left(\frac{dz}{du}\right)^2\right\} = 2u^{-4} + 20a_4u^{-6} + 42a_6u^{-8} + 72a_8u^{-10} + 110a_{10}u^{-12} + \ldots.$$

Also $$\frac{2}{u^2(1+u^2)} = 2u^{-4} - 2u^{-6} + 2u^{-8} - 2u^{-10} + 2u^{-12} + \ldots.$$

Hence subtracting

$$2\left\{z\frac{d^2z}{du^2} + \left(\frac{dz}{du}\right)^2 - \frac{1}{u^2(1+u^2)}\right\} = (20a_4 + 2)u^{-6} + (42a_6 - 2)u^{-8} + (72a_8 + 2)u^{-10} + (110a_{10} - 2)u^{-12} + \ldots.$$

And since the differential equation gives

$$2z^2\left\{z\frac{d^2z}{du^2} + \left(\frac{dz}{du}\right)^2 - \frac{1}{u^2(1+u^2)}\right\} = \left(2z\frac{dz}{du}\right)^2$$

we have

$$\{A + \tfrac{1}{3}r^2 + a_4r^4 + a_6r^6 + a_8r^8 + a_{10}r^{10} + \ldots.\} \times \{(20a_4 + 2)r^6 + (42a_6 - 2)r^8 + (72a_8 + 2)r^{10} + (110a_{10} - 2)r^{12} + \ldots.\} = (\tfrac{2}{3}r^3 + 4a_4r^5 + 6a_6r^7 + 8a_8r^9 + 10a_{10}r^{11} + \ldots.)^2$$

where r is now substituted for u^{-1}.

Equating coefficients,

$$A(20a_4 + 2) = \frac{4}{9},$$

$$A(42a_6 - 2) + \frac{1}{3}(20a_4 + 2) = \frac{16}{3}a_4,$$

$$A(72a_8 + 2) + \frac{1}{3}(42a_6 - 2) + a_4(20a_4 + 2) = 8a_6 + 16a_4^2,$$

and so on.

Hence $$a_4 = -\frac{1}{10}\left(1 - \frac{2}{9A}\right),$$

$$a_6 = +\frac{1}{21}\left(1 - \frac{4}{15A} - \frac{2}{135A^2}\right),$$

$$a_8 = -\frac{1}{36}\left(1 - \frac{142}{525A} - \frac{13}{525A^2} - \frac{16}{14175A^3}\right).$$

The calculation of the higher coefficients becomes rather laborious.

For small values of r the potential is accordingly given by

$$\phi = -\frac{1}{2}\log\left\{A + \frac{1}{3}r^2 - \frac{1}{10}\left(1 - \frac{2}{9A}\right)r^4 + \frac{1}{21}\left(1 - \frac{4}{15A} - \frac{2}{135A^2}\right)r^6 - \frac{1}{36}\left(1 - \frac{142}{525A} - \frac{13}{525A^2} - \frac{16}{14175A^3}\right)r^8 + \ldots\right\}$$

8. It is desirable to form some idea of the magnitude of the constant A. This depends on the density of the system, for at the centre $A = 1/4\pi\sigma$. Our units have been chosen so that the distance of the sun from the centre of the system is about $\sqrt{3}$ units of distance, and the unit of velocity is equal to $\sqrt{\pi}$ times the mean velocity of the stars in the radial direction. The velocity-unit is thus about 50 km. per sec. If we take (for example) the unit of distance to be 100 parsecs, the unit of density is found to be equivalent to 7000 times the sun's mass in a sphere of 5 parsecs radius. (The unit of density varies inversely as the square of the unit of length, the unit of velocity being kept constant.) A lower limit to the density in the neighbourhood of the sun may be taken to be 10 times the sun's mass in a sphere of 5 parsecs radius, *i.e.* 1/700 unit, and the density at the centre must be at least 4 times this owing to the factor $(1 + r^2)$ in (19). We cannot assign any upper limit, since we do not know the number of dark stars. Working with the lower limit we find the constant $A = 14$. It can only be increased by taking the distance of the sun from the centre of the system to be less than $100\sqrt{3}$ parsecs; whereas it may be diminished if the sun's distance is greater than this, or if there are many dark stars uncounted in our estimate of density, or by the increase in the factor z^2 between the centre and the sun. It appears probable therefore that A will be considerably less than 14 —perhaps less than unity. This is rather unfortunate, because when A is large the series reduces to the simple, though not very convergent, form

$$z^2 = A + \frac{2}{2.3}r^2 - \frac{2}{4.5}r^4 + \frac{2}{6.7}r^6 - \frac{2}{8.9}r^8 + \ldots.$$

In order to obtain information as to the nature of the law of density and potential, I have worked out the case when $A = 1$. In the following table the results for $r = 0{\cdot}5$ have been derived from the series already obtained. From $r = 0{\cdot}5$ to $r = 1{\cdot}0$ they

have been derived by quadratures, using the differential equation. From $r = 1{\cdot}0$ to $r = 2{\cdot}0$ the results have been continued by a Taylor series, the coefficients being obtained by successive differentiations of the differential equation.

Solution for $A = 1$.

r.	$4\pi\sigma$.	ϕ.	z^2.	$d(z^2)/du$.	u.
0	1·000	·000	1·000	·000	∞
0·5	0·741	−·038	1·079	−·075	2·0
0·625	0·642	−·057	1·120	−·139	1·6
0·833	0·491	−·092	1·202	−·298	1·2
1·0	0·391	−·123	1·278	−·474	1·0
1·25	0·279	−·169	1·401	...	0·8
1·5	0·201	−·213	1·53	...	0·67
1·75	0·147	−·256	1·67	...	0·57
2·0	0·111	−·294	1·80	...	0·5

The first three columns give the distance from the centre, relative density, and potential; the remaining columns give auxiliary quantities used in the calculations.

The table has been carried far enough to include the sun (near $r = 1{\cdot}75$). We see that the central density is about 7 times that in our own neighbourhood. The value of the potential $\phi = -{\cdot}256$ may be interpreted to mean that a star would acquire a velocity of 36 km. per sec. in falling from the sun to the centre of the system. If we take the density near the sun to be 10 times the sun's mass in a sphere of 5 parsecs radius, the unit of distance becomes 280 parsecs, so that the sun's distance is nearly 500 parsecs. To multiply the density fourfold would halve the distance, and so on. It is, however, necessary to assume the sun's distance from the centre to be considerable, for otherwise the density in the outer parts of the system would fall off much more rapidly than the observations indicate. Owing to the factor $(1 + r^2)$ in the density, this is true whatever value we assume for A; thus at 3 times the sun's distance the density cannot be more than 1/7 of that near the sun. (By a rather risky extrapolation it is found to be about 1/14.) It follows that the stellar density near the sun cannot be taken much greater than the lower limit we have adopted.

9. An interesting result derived from this investigation is the change of the velocity-ellipsoid at different distances from the centre. At the centre the ellipsoid is a sphere; proceeding outwards, the major axis remains the same, but the minor axis diminishes continually according to the law $(h^2 + k^2r^2)^{-\frac{1}{2}}$.

This result could probably be tested by observation, for our data certainly reach to regions in which the prolateness of the velocity-ellipsoid must have values which differ considerably. The law of density found, and the fact that the major axes of the

ellipsoids in different parts are not parallel but converge towards the centre, could probably also be tested to a certain extent. But the theory at present has dealt only with a globular system, and I am not without hope that it may be possible to determine the modifications required by the actual oblate form of the stellar system. Any laborious comparison with observation is perhaps better deferred until the theory has been further generalised.

Summary.

1. Systems having spherical symmetry have been discussed in which the distribution of stellar velocities everywhere satisfies Schwarzschild's ellipsoidal hypothesis. The prolateness of the velocity-ellipsoid may depend on the distance from the centre.

2. In all such systems the potential must satisfy the differential equation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \frac{e^{2\phi}}{1+r^2} = 0.$$

The density is then given by $4\pi\sigma = e^{2\phi}/(1+r^2)$.

3. Neglecting the possibility of systems with a hollow centre, we find that there is one singular solution with the density infinite at the centre, but that in general the central density is finite and not excessively greater than that near the sun. The central density cannot, however, be less than 4 times that near the sun.

4. The prolateness of the velocity-ellipsoid must increase with increasing distance from the centre. The major axis remains constant, and the minor axis (which is equal to it at the centre) continually decreases outwards. The axes are equal to $1/h$ and $1/(h^2+k^2r^2)^{\frac{1}{2}}$, where h and k are constants.

5. A particular case is given by way of illustration, with the results

central density = 7 times density near the sun,
velocity acquired in falling from the sun to the centre = 36 km. per sec.

If the density near the sun is equal to 10 times the sun's mass in a sphere of 5 parsecs radius, the sun's distance from the centre is 500 parsecs. If the density is greater, the distance is reduced; but to reduce it very much would lead to a contradiction with observation as regards the relative density in the outer parts of the system.

6. In comparing this specialised problem with that presented by the actual universe, it is probable that the oblate shape of the latter may introduce some important differences.

The Secular Acceleration of the Moon's Mean Motion as determined from the Occultations in the Almagest. By J. K. Fotheringham, M.A., D.Litt., assisted by Gertrude Longbottom, M.A.

Although numerous attempts have been made to determine the acceleration of the Moon's mean motion by means of ancient eclipses both of the Sun and of the Moon, I am not aware that the ancient occultations have ever been discussed. The reason for this is, probably, that the eclipses have been regarded as more satisfactory material. On the other hand, it may be urged that the eclipses do not give us the sidereal motion of the Moon, but only the difference between her motion and that of the Sun, and, with less certainty, the difference between the motion of the Sun and that of the Moon's node. We cannot discover the sidereal motion of the Moon from eclipses only, unless we assume either that the secular acceleration of the Sun's mean motion is negligible or that the theoretical acceleration of the motion of the node does not admit of any empirical correction. Of these two assumptions, the one has been assailed by Dr. Cowell,* the other by Dr. Brown,† and this renders it the more important that the ancient occultations, which compare the Moon directly with the stars, should be investigated.

Newcomb in his *Researches on the Motion of the Moon*, part i. (Washington, 1878), p. 279, gives reason for neglecting the ancient occultations. I quote his argument verbatim :—

"The occultations cited by Ptolemy seemed worthy of more confidence, and I went so far as to make a summary of Ptolemy's statements; but they were so far from being precise that some hesitancy was felt in deciding whether they were worth employing. I finally concluded to omit them, from the fact that they are cited by Ptolemy, not for the purpose of presenting a complete series of observations, but to prove that the erroneous value of the constant of precession deduced by Hipparchus, namely, one degree in a century, was correct. Being in all probability selected for the purpose of proving an erroneous hypothesis, it was hardly possible to employ them without the reservation above alluded to, that they should be accepted or rejected according as their results did or did not agree with those derived from other data."

This paragraph contains two specific objections to the use of the occultations, (1) that they are not precise, (2) that they were used by Ptolemy to prove an erroneous hypothesis, and may have been selected because they supported that hypothesis.

To the first of these objections I should reply that the occultations are more precisely recorded than the solar eclipses which have been used in most investigations of secular acceleration, though they are all rejected by Newcomb, and that, if they are

* *Monthly Notices*, lxvi. (1905), pp. 3–5, and elsewhere.
† *The Observatory*, xxxvii. (1914), p. 207.

inferior in accuracy to the lunar eclipses, they cannot be rejected unless it is known that the lunar eclipses are capable of giving the Moon's position in relation to the stars; but, as I have shown above, this is far from being universally admitted. Moreover, Ptolemy appears to give us the records of occultations much as he found them in Timocharis, Agrippa, and Menelaus, whereas the lunar eclipses are mainly quoted through the medium of Hipparchus, and, as they are regularly expressed in equinoctial time, had probably undergone some reduction before the form in which they are given in the Almagest was obtained. The one eclipse of which a cuneiform record has come down to us (that of -522 July 16)* was certainly very loosely reduced, and I should prefer to reject the whole of the Babylonian eclipses and confine the discussion of ancient lunar eclipses to those observed at Alexandria and Rhodes.

Newcomb's second objection is easily met. Hipparchus had obtained a defective value for the tropical year from equinoctial and solstitial observations which we know to have been vitiated by refraction; at the same time, he derived or confirmed from lunar eclipses a value for the synodic month as good as modern science could find for him. In this way he carried over to the motion of the Moon in longitude the error of his equinoctial and solstitial observations. His value, which Ptolemy adopts, for the mean motion of the Moon in longitude, expressed in modern terminology, is $1732562497''$ per Julian century. The mean epoch of the occultations in the Almagest is -123. For this epoch I find in the present inquiry a mean motion of $1732563948''$ per century. Hansen's tables would give rather more, Newcomb's rather less. From this it will be seen that Ptolemy underestimated the mean motion of the Moon in longitude by about $1451''$ per Julian century. Ptolemy proceeds to use the tabular places of the Moon as determined with this erroneous value to find the changes in longitude of stars occulted by the Moon. Had the occultations and his discussion been perfect, he would have transferred to precession the error of his lunar tables, and would have obtained for precession a value less by $1451''$ per century than the correct value. As the correct value for precession at the epoch in question was $+4981''$ per century, he ought to have obtained $+3530''$ per century. As a fact, he confirms Hipparchus's minimum value of one degree per century. The difference is trifling, and it follows that the occultations have done what good observations ought to have done. The error in the result is entirely due to the error in the tables used in the discussion. We may therefore acquit Ptolemy of the charge of having selected observations specially adapted to the purpose of proving an erroneous hypothesis.

I come now to the text of the records of occultations, which I arrange chronologically, and to each of which I append an English

* For a full account of the cuneiform record, see Kugler, *Sternkunde und Sterndienst in Babel,* i. (1907), pp. 61–75.

translation. They are all to be found in the third chapter of the seventh book of the Almagest.

(1) Ed. Heiberg, p. 32. *πάλιν Τιμόχαρις μέν φησιν ἐν Ἀλεξανδρείᾳ τηρήσας, ὅτι τῷ λς΄ ἔτει τῆς πρώτης κατὰ Κάλιππον περιόδου τοῦ μὲν Ποσειδεῶνος τῇ κέ, τοῦ δὲ Φαωφὶ τῇ ις΄, ὥρας ι΄ ἀρχούσης ἀκριβῶς σφόδρα ἐφαίνετο κατειληφυῖα ἡ σελήνη τῇ βορείῳ ἀψῖδι τὸν πρὸς ἄρκτον τῶν ἐν τῷ μετώπῳ τοῦ Σκορπίου.*

"Further, Timocharis records the following observation made by him at Alexandria:—In the 36th year of the 1st Calippic cycle, on the 25th day of Poseideon and the 16th of Phaophi, at the very beginning of the 10th hour, the Moon was observed to have overtaken with her north cusp the most northerly of the stars on the front of the Scorpion."

(2) *Ibid.*, p. 28. *πάλιν Τιμόχαρις μὲν ἀναγράφει τηρήσας ἐν Ἀλεξανδρείᾳ, διότι τῷ λς΄ ἔτει τῆς πρώτης κατὰ Κάλιππον περιόδου τοῦ μὲν Ἐλαφηβολιῶνος τῇ ιέ, τοῦ δὲ Τυβὶ τῇ έ, ὥρας γ΄ ἀρχομένης ἡ σελήνη μέση τῇ πρὸς ἰσημερινὴν ἀνατολὴν ἀψῖδι τὸν Στάχυν κατέλαβεν, καὶ διῆλθεν ὁ Στάχυς ἀφαιρῶν αὐτῆς τῆς διαμέτρου πρὸς ἄρκτους τὸ τρίτον μέρος ἀκριβῶς.*

"Further, Timocharis records the following observation made by him at Alexandria:—In the 36th year of the 1st Calippic cycle, on the 15th day of Elaphebolion and the 5th of Tybi, as the 3rd hour was beginning, the Moon, with the middle of her disk on the exact east, overtook Spica, and Spica passed through her, cutting off one-third of her diameter exactly, on the north side."

(3) *Ibid.*, p. 25. *Τιμόχαρις μὲν γὰρ ἀναγράφει τηρήσας ἐν Ἀλεξανδρείᾳ ταῦτα, διότι τῷ μζ΄ ἔτει τῆς πρώτης κατὰ Κάλιππον ἐξκαιεβδομηκονταετηρίδος τῇ ή τοῦ Ἀνθεστηριῶνος, κατ' Αἰγυπτίους τῇ κθ΄ τοῦ Ἀθύρ, ὥρας γ΄ ληγούσης τὸ νότιον μέρος ἥμισυ τῆς σελήνης ἐπιβεβηκὸς ἐφαίνετο ἐπὶ τὸ ἑπόμενον ἤτοι γ΄ ἢ Ι΄ μέρος τῆς Πλειάδος ἀκριβῶς.*

"For Timocharis records the following observation made by him at Alexandria:—In the 47th year of the 1st Calippic cycle of 76 years, on the 8th day of Anthesterion, the 29th of Athyr of the Egyptian calendar, as the 3rd hour was ending, the southern half of the Moon was observed to have covered the eastern third part or half of the Pleiades exactly.

(4) *Ibid.*, p. 29. *Καὶ ἐν τῷ μή δὲ ἔτει τῆς αὐτῆς περιόδου φησὶν ὁμοίως, ὅτι τοῦ μὲν Πυανεψιῶνος τῇ ς΄ φθίνοντος, τοῦ δὲ Θὼθ τῇ ζ΄, τῆς ι΄ ὥρας ὅσον ἡμιωρίου προελθόντος ἐκ τοῦ ὁρίζοντος ἀνατεταλκυίας τῆς σελήνης ὁ Στάχυς ἐφαίνετο ἁπτόμενος αὐτοῦ τοῦ βορείου ἀκριβῶς.*

"And in the 48th year of the same cycle, he says, in like manner, that on the 6th day before the end of Pyanepsion, the 7th day of Thoth, when the 10th hour had lasted half an hour, when the Moon had risen from the horizon, Spica was observed exactly touching her north limb."

(5) *Ibid.*, p. 27. *Ἀγρίππας δ' ἐν Βιθυνίᾳ τηρήσας ἀναγράφει, ὅτι τῷ ιβ΄ ἔτει Δομετιανοῦ κατ' αὐτοὺς Μητρῴου ζ΄ νυκτὸς ὥρας γ΄ ἀρχούσης ἡ σελήνη ἐπεκάλυψε τῷ νοτίῳ κέρατι τὸ ἑπόμενον καὶ νότιον μέρος τῆς Πλειάδος.*

"And Agrippa records the following observation made by him in Bithynia:—In the 12th year of Domitian, on the 7th day of Metrous in the local calendar, as the 3rd hour of the night was beginning, the Moon with her south cusp occulted the south-east part of the Pleiades."

(6) *Ibid.*, p. 30. Μενέλαος δὲ ὁ γεωμέτρης ἐν Ῥώμῃ φησὶν τετηρῆσθαι τῷ α΄ ἔτει Τραιανοῦ Μεχὶρ ιε΄ εἰς τὴν ις΄ ὥρας ι΄ πεπληρωμένης τὸν Στάχυν ὑπὸ τῆς σελήνης ἠφανισμένον· μὴ ὁρᾶσθαι γάρ· ἀλλ᾽ ὥρας ἑνδεκάτης ληγούσης τεθεωρῆσθαι προηγούμενον τοῦ κέντρου τῆς σελήνης ἔλαττον τῆς διαμέτρου αὐτῆς ἴσον ἀπέχοντα τῶν κεραιῶν.

"And Menelaus, the geometrician, states that at Rome in the 1st year of Trajan, on the night of Mechir 15–16, when the 10th hour had been completed, Spica was observed to be in occultation by the Moon; for it was invisible; but as the 11th hour was ending, it was observed preceding (*i.e.* to the west of) the centre of the Moon by a distance of less than the Moon's diameter, being equidistant from the two cusps."

(7) *Ibid.*, p. 33. Μενέλαος δὲ ὁμοίως ἐν Ῥώμῃ τηρήσας φησίν, ὅτι τῷ α΄ ἔτει Τραιανοῦ Μεχὶρ ιή εἰς τὴν ιθ΄ ὥρας ια΄ ληγούσης ἐφαίνετο ἐπ᾽ εὐθείας τῷ τε μέσῳ καὶ τῷ νοτίῳ τῶν ἐν τῷ μετώπῳ τοῦ Σκορπίου ἡ νότιος κεραία τῆς σελήνης, τὸ δὲ κέντρον αὐτῆς ὑπελείπετο τῆς εὐθείας καὶ τοσοῦτον ἀπεῖχεν ἀπὸ τοῦ μέσου, ὅσον ὁ μέσος ἀπὸ τοῦ νοτίου, ἐδόκει δὲ κατειληφέναι τὸν βόρειον τῶν ἐν τῷ μετώπῳ· οὐδαμοῦ γὰρ ἐφαίνετο.

"Menelaus in like manner records the following observation made by him at Rome:—In the first year of Trajan, on the night of Mechir 18–19, as the 11th hour was ending, the south cusp of the Moon was seen in a straight line with the middle and the most southerly of the stars on the front of the Scorpion, but her centre was to the east of the straight line and as far distant from the middle star as the middle star was from the southern, and she appeared to have occulted the northernmost of the stars on the front, for it was nowhere to be seen."

In the discussion of each of these observations, Ptolemy reduces the date to the era of Nabonassar and, where necessary, to the Egyptian vague year. I have adopted his reductions without question. He also understands the time to be expressed in each case in "seasonal" hours, *i.e.* in hours each of which comprises one-twelfth of the interval from sunset to sunrise, and which therefore vary in length according to the season of the year. It will be observed that, as in the case of the lunar eclipses, the hour seems to be ordinarily divided into three parts, ὥρας ἀρχούσης, ἡμιωρίου προελθόντος, and ὥρας ληγούσης. I have computed on each occasion for the middle of the third part indicated, except in the case of (1), where the time is qualified by the words ἀκριβῶς σφόδρα, which I take to indicate the exact beginning of the seasonal hour, and in the case of (6), where I take πεπληρωμένης to indicate the exact end of the seasonal hour.

The computation has been made as follows:—

The solar places were found by means of Neugebauer's

Abgekürzte Tafeln der Sonne und der grossen Planeten,* into which I introduced the correction $\Delta\pi = -0^{\circ}\cdot17$ for the first four observations, and $\Delta\pi = -0^{\circ}\cdot13$ for the remaining observations, in order to bring the tables into agreement with Newcomb's value for the motion of the solar perigee. The times of sunrise and sunset were next obtained, the latitude and longitude of Bithynia being taken as 40° 35′ N., 29° 47′ E., and the true zenith distance of Sun at sunrise and sunset being taken in each case as 90° 48′. The times of observation were then obtained by interpolation.

The lunar places for each observation were then found. For this purpose the following elements were used:—

$$g = 110^{\circ}\ 19'\ 38'' + 171791\ 5794''\,T + 46''\cdot1\,T^2 + 0''\cdot052\,T^3$$
$$\omega = 192\ \ 7\ 25 + \ \ 2161\ 1516\,T - 44''\cdot5\,T^2 - 0''\cdot050\,T^3$$
$$\Omega = \ 33\ 16\ 21 - \ \ 696\ 2921\,T + \ 7''\cdot5\,T^2 + 0''\cdot006\,T^3$$

where T represents the number of Julian centuries elapsed since 1800 January 0·0 G.M.T.

In these formulæ the places at epoch and coefficients of T are from Dr. Cowell's paper (*M.N.*, lxv. (1905), p. 863), the coefficients of T^2 for ω and Ω are taken from Dr. Brown (*M.N.*, lxx. (1909), p. 148), and the coefficients of T^3 are taken from Newcomb's paper (*M.N.*, lxix. (1909), p. 461). The coefficient of T^2 for g is chosen so as to give a value of $+8''$ per century to the sidereal secular acceleration of the Moon. This is Newcomb's latest value.

The true longitude, latitude, and parallax of the Moon for each observation were then found by applying the inequalities tabulated in Dr. Cowell's paper (*M.N.*, lxv. p. 864), the coefficient of sin $-g'$ being subjected to a correction of $-0''\cdot0166\,t$ to give effect to the secular diminution of the eccentricity of the Earth's orbit,† where t is the number of Julian years elapsed since 1850. For the mean equatorial horizontal parallax the value 3423″ has been adopted. Planetary and figure of the Earth terms were introduced as an exercise into the work on the first observation. Their effect on the true longitude was $-8''$. They are of no importance for the present inquiry.

The apparent longitude, latitude, and semi-diameter of the Moon were then found by the formulæ given in Newcomb's *Researches on the Motion of the Moon*, part i. p. 66, which are repeated in part ii. of the same work (Washington, 1912), pp. 18, 19. For this purpose the obliquity of the ecliptic was taken from Newcomb's *Tables of the Sun*,‡ and s' was taken as equal to $\frac{[4\cdot74979]}{R}\sin\pi$, corresponding to a value of 0·2725 for k.

* *Veröffentlichungen der königlichen astronomischen Rechen-Instituts zu Berlin*, No. 25 (1904), now superseded by the same author's *Tafeln für Sonne, Planeten, und Mond* (1914).

† A similar correction is made in the *Tables de la Lune*, published by the Bureau des Longitudes (1911), where, however, Newcomb's value for the diminution at epoch 1900 has been erroneously applied to epoch 1850.

‡ *Astronomical Papers printed for the use of the American Ephemeris*, vol. vi. part i. (1895).

The longitude, latitude, and semi-diameter were then differentiated both for the secular acceleration and for the time.

For the true longitude we have used :—

$$\frac{dl}{ds_L} = +1''T^2 + 0''\cdot110T^2\cos g + 0''\cdot023T^2\cos 2D + 0''\cdot022T^2\cos(-g+2D) + 0''\cdot007T^2\cos 2g$$

$$\frac{dl}{dt} = +474'' + 52''\cos g + 10''\cos 2D + 9''\cos(-g+2D) + 4''\cos 2g - 2''\cos(-2F),$$

where s_L is the centennial secular acceleration expressed in seconds of arc and t is the time expressed in units of 0·01 days.

For the true latitude we have used :—

$$\frac{db}{ds_L} = +0''\cdot0895T^2\cos F + 0''\cdot0098T^2\cos(g+F) + 0''\cdot0030T^2\cos(-F+2D) + 0''\cdot0019T^2\cos(-g+F+2D) + 0''\cdot0017T^2\cos(F+2D) + 0''\cdot0009T^2\cos(2g+F)$$

$$\frac{db}{dt} = +42''\cdot6\cos F + 4''\cdot6\cos(g+F) + 1''\cdot2\cos(-F+2D) + 0''\cdot9\cos(-g+F+2D) + 0''\cdot8\cos(F+2D).$$

For the apparent places we have used :—

$$\frac{dl'}{ds_L} = \left\{1 + p\cos(l-\lambda)\right\}\frac{dl}{ds_L}$$

$$\frac{db'}{ds_L} = \frac{db}{ds_L},$$

where p and λ are taken from the formula used in finding the apparent place of the Moon.

For the differentiation as to the time we have taken :—

$$\frac{d\tau}{dt} = +12995''$$

$$\cos E = \sin\omega\cos\tau\sec\beta$$

$$\frac{d\lambda}{dt} = \frac{d\tau}{dt}\cos\phi'\sin E\sec\beta$$

$$\frac{dp}{dt} = \frac{d\tau}{dt}q\cos\phi'\cos E$$

$$\frac{dl'}{dt} = \frac{dl}{dt} + p\cos(l-\lambda)\left\{\frac{dl}{dt} - \frac{d\lambda}{dt}\right\} + \frac{dp}{dt}\sin(l-\lambda)$$

$$\frac{dq}{dt} = -\frac{d\tau}{dt}p\cos\phi'\cos E$$

$$\frac{dR}{dt} = -\frac{dp}{dt}\cos b\cos(l-\lambda) - \frac{dq}{dt}\sin b + p\cos b\sin(l-\lambda)\left\{\frac{dl}{dt} - \frac{d\lambda}{dt}\right\}$$

$$\frac{db'}{dt} = \frac{db}{dt} - \frac{dq}{dt} - \frac{dR}{dt}\left(\frac{\sin(b-q)}{R^2}\right).$$

For the meanings of the terms used in this differentiation, see the formula of Newcomb's by which the apparent place of the Moon was obtained.

$$\frac{ds'}{ds_1} = 0$$

$$\frac{ds'}{dt} = s' \frac{dR}{dt},$$ where $\frac{dR}{dt}$ is expressed numerically, and is therefore to be obtained by multiplying by sin 1″ the value given above.

The star places were next found as follows:—

The proper motions in Right Ascension and Declination for 1875 were taken from Boss's *Preliminary General Catalogue* (1910). To these were applied Boss's corrections by the formula given on p. xxviii of that work. The proper motions were then converted to longitude and latitude by the formula given in Newcomb's *Researches*, part i. p. 58:—

$$\cos E = \sin \omega \cos \alpha \sec B$$
$$\mu_1 = \mu \sin E \cos \delta \sec B + \mu' \cos E \sec B$$
$$\mu_2 = -\mu \cos E \cos \delta + \mu' \sin E.$$

These proper motions were assumed to be constant, and were applied to the longitudes and latitudes given in Newcomb's *Researches*, part i. pp. 203, 204, or sometimes to the newer and more complete list in part ii. pp. 78–84. The difference between the two sets of places is negligible. In this way the places at the dates of observation, referred to the equinox of 1850, were found.

The reduction for precession was then made by the following formulæ:—

$$\lambda = \lambda_0 - 47''{\cdot}144\mathrm{T} \tan \beta_0 \cos (\lambda_0 + 6^\circ\ 32'{\cdot}21) + 5025''{\cdot}05\mathrm{T} + 0''{\cdot}202\mathrm{T}^2 \tan \beta_0 \sin (\lambda_0 + 16^\circ{\cdot}2) + 1''{\cdot}1125\mathrm{T}^2$$
$$\beta = \beta_0 + 47''{\cdot}144\mathrm{T} \sin (\lambda_0 + 6^\circ\ 32'{\cdot}21) + 0''{\cdot}202\mathrm{T}^2 \cos (\lambda_0 + 16^\circ{\cdot}2),$$

where λ, β are true longitude and latitude respectively referred to equinox of date, λ_0 and β_0 are the same referred to equinox of 1850, and T is the interval elapsed in solar centuries since 1850. The formulæ are derived from Newcomb's *Compendium of Spherical Astronomy* (1906), pp. 287, 288, but the numerical values have been adjusted so as to conform to Boss's corrections. In the T^2 term in λ "sin" is misprinted "cos" in Newcomb's *Compendium*. This error is corrected in his *Researches*, part ii. p. 19.

The apparent place of each star was then found by applying the correction,

$$\Delta\lambda = -20''{\cdot}52 \cos (\odot - \lambda) \sec \beta$$
$$\Delta\beta = -20''{\cdot}52 \sin (\odot - \lambda) \sin \beta.$$

When the computation had been completed, diagrams were drawn showing the respective positions of Moon and stars, and the amount and direction of corrections to the Moon's position for secular acceleration and time. Where necessary the positions of the cusps were found.

The principal stages in the computation were as follows:—

	I.	II.	III.	IV.	V.	VI*a*.	VI*b*.	VII.
Era of Nabonassar	454 Phao. 16 7	454 Tyb. 5–6	465 Ath. 29–30	466 Tho. 7–8	840 Tyb. 2–3	845 Mec. 15–6	845 Mec. 15–6	845 Mec. 18–9
Julian date	−294 Dec. 20	−293 Mar. 9	−282 Jan. 29	−282 Nov. 8	+92 Nov. 29	+98 Jan. 10	+98 Jan. 10	+98 Jan. 13
Place	Alexandria	Alexandria	Alexandria	Alexandria	Bithynia	Rome	Rome	Rome
Longitude	29°·84	29°·84	29°·84	29°·84	29°·79	12°·46	12°·46	12°·46
Latitude	31° 11′	31° 11′	31° 11′	31° 11′	40° 35′	41° 53′	41° 53′	41° 53′
Berlin time of sunset	59°·51	73°·93	66°·70	61°·49	52°·90	73°·55	73°·55	74°·34
,, sunrise	268°·23	259°·53	268°·72	259°·27	270°·49	294°·08	294°·08	293°·80
G.M.T. of observation	$13^h\ 31^m$	$6^h\ 16^m$	$6^h\ 44^m$	$13^h\ 39^m$	$5^h\ 15^m$	$16^h\ 16^m$	$17^h\ 17^m$	$17^h\ 16^m$
True longitude of Sun	265°·49	344°·32	306°·01	222°·75	247°·01	290°·04	290°·08	293°·07
Mean anomaly ,,	21°·06	98°·64	60°·35	339°·57	356°·83	38°·42	38°·46	41°·42
,, longitude of Moon	210° 40′ 31″	167° 37′ 47″	36° 45′ 23″	169° 28′ 23″	29° 41′ 52″	169° 15′ 6″	169° 48′ 32″	209° 19′ 44″
,, anomaly ,,	194° 52′ 53″	143° 4′ 5″	288° 52′ 6″	30° 1′ 20″	148° 13′ 17″	79° 36′ 9″	80° 9′ 19″	119° 23′ 47″
,, elongation ,,	305°·93	185°·30	92°·53	306°·00	142°·58	240°·47	240°·99	277°·55
,, argument of latitude	14° 54′ 18″	336° 1′ 38″	55° 52′ 4″	203° 35′ 12″	98° 55′ 8″	337° 25′ 7″	337° 58′ 41″	17° 39′ 25″
True longitude of Moon	209° 32′ 3″	170° 25′ 18″	29° 7′ 16″	171° 13′ 9″	33° 7′ 59″	176° 54′ 7″	177° 27′ 54″	215° 32′ 6′
,, latitude ,,	+1° 6′ 33″	−1° 46′ 38″	+3° 59′ 45″	−2° 5′ 12″	+4° 59′ 38″	−1° 15′ 19″	−1° 12′ 26″	+2° 4′ 8″
H.E. parallax	54′ 21″	54′ 41″	57′ 20″	59′ 16″	54′ 11″	57′ 39″	57′ 36″	55′ 7″
Apparent longitude of Moon	210° 23′ 58″	171° 15′ 4″	28° 24′ 33″	172° 9′ 57′	33° 21′ 16″	177° 3′ 42″	177° 27′ 41″	215° 40′ 33′
,, latitude ,,	+0° 53′ 47″	−1° 54′ 59″	+3° 54′ 29″	−2° 13′ 31″	+4° 29′ 37″	−1° 53′ 23″	−1° 55′ 13″	+1° 20′ 21″
Semi-diameter of Moon	14′ 51″	14′ 59″	15′ 48″	16′ 14″	14′ 57″	15′ 54″	15′ 53″	15′ 10″
$\frac{dl'}{ds_L}$	+398″	+406″	+439″	+468″	+262″	+298″	+298″	+270″
$\frac{db'}{ds^L}$	+ 34″	+ 34″	+ 26″	− 37″	− 6″	+ 25″	+ 26″	+ 21″
$\frac{dl'}{dt}$	+385″	+367″	+355″	+ 459′	+322″	+340″	+349″	+296″
$\frac{db'}{dt}$	−12″·9	+12″·6	+38″·0	−58″·4	+59″·5	−22″·6	−17″·2	−11″
$\frac{ds'}{dt}$	−0″·68	−0″·67	−0″·60	−0″·83	−0″·35	+0″·12	+0″·30	+0″·12

Longitude of north cusp	210° 24′ 8″	...	...	172° 9′ 26″	33° 19′ 31″	...	177° 27′ 30″	
Latitude ,,	+1° 8′ 38″	...	...	−1° 57′ 18″	+4° 44′ 27″	...	−1° 39′ 48″	
Longitude of south cusp	...	...	...	...	33° 23′ 1″	...	...	215° 40′ 28″
Latitude ,,	...	...	...	...	+4° 14′ 47″	...	...	+1° 5′ 11″
Apparent longitude of β Scorpii	211° 19′ 11″	...	...	...	...	...	...	216° 44′ 1″
,, latitude of β Scorpii	+1° 17′ 53″	...	..	...	...	...	...	+1° 16′ 20″
,, longitude of Spica	...	172° 0′ 30″	...	172° 9′ 37″	...	177° 24′ 34″	177° 24′ 34″	
,, latitude ,,	...	−1° 54′ 11″	...	−1° 54′ 13″	...	−1° 55′ 32″	−1° 55′ 32″	
,, longitude of η Tauri	...	...	28° 17′ 7″	...	33° 28′ 33″			
,, latitude ,,	...	...	+3° 49′ 19″	...	+3° 51′ 30″			
,, longitude of 17 Tauri	...	...	27° 42′ 16″	...	32° 53′ 46″			
,, latitude ,,	...	...	+3° 57′ 50″	...	+3° 59′ 59″			
,, longitude of 19 ,,	...	...	27° 51′ 50″	...	33° 3′ 11″			
,, latitude ,,	...	...	+4° 17′ 21″	...	+4° 19′ 32″			
,, longitude of 20 ,,	...	...	27° 58′ 24″	...	33° 9′ 35″			
,, latitude ,,	...	...	+4° 9′ 37″	...	+4° 11′ 52″			
,, longitude of 23 ,,	...	...	27° 59′ 34″	...	33° 10′ 52″			
,, latitude ,,	...	...	+3° 43′ 52″	...	+3° 46′ 2″			
,, longitude of 27 ,,	...	...	28° 38′ 55″	...	33° 50′ 21″			
,, latitude ,,	...	...	+3° 41′ 20″	...	+3° 43′ 30″			
,, longitude of 28 ,,	...	...	28° 40′ 20″	...	33° 51′ 46″			
,, latitude ,,	...	...	+3° 46′ 5″	...	+3° 48′ 17″			
,, longitude of δ Scorpii	...	...	...	...	...	...	...	216° 7′ 15″
,, latitude ,,	...	...	...	...	...	..	...	−1° 43′ 3″
,, longitude of π ,,	...	...	...	...	...	...	...	216° 29′ 45″
,, latitude ,,	...	...	...	...	..	...	...	−5° 12′ 23″

These positions are shown on the appended diagrams, along with the effect of corrections to secular acceleration and time of observation. Two diagrams have been drawn to represent the

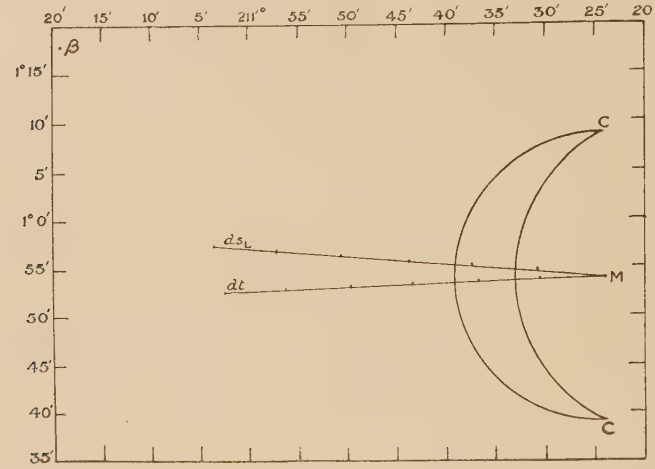

FIG. 1.

two stages of the sixth observation, while two, varying in scale, have also been drawn for the seventh observation.

I now discuss the observations in detail.

(i.) It is clear from the diagram that in this case the Moon

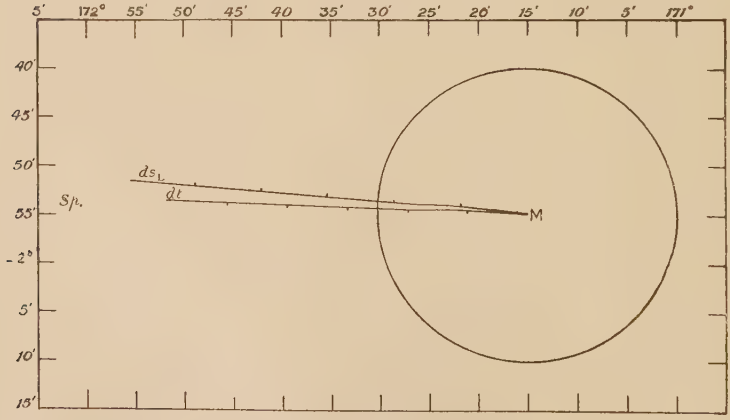

FIG. 2.

just failed to occult β Scorpii. The record does not, however, assert an actual occultation, but merely that the Moon caught the star with her north cusp. The greater the correction to secular acceleration, the closer the cusp passes to the star. I have assumed

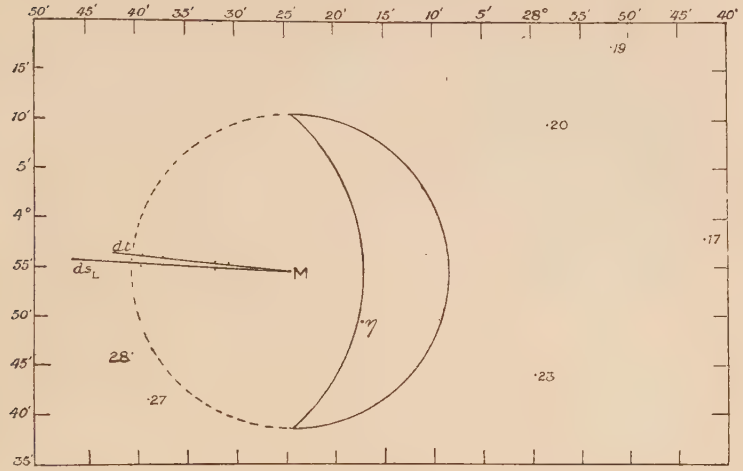
50′ 45′ 40′ 35′ 30′ 25′ 20′ 15′ 10′ 5′ 28° 55′ 50′ 45′ 40′
·19
15′
10′
·20
5′
4°
·17
55′
dt
ds_L
M
·η
50′
28·
45′
·23
·27
40′
35′

FIG. 3.

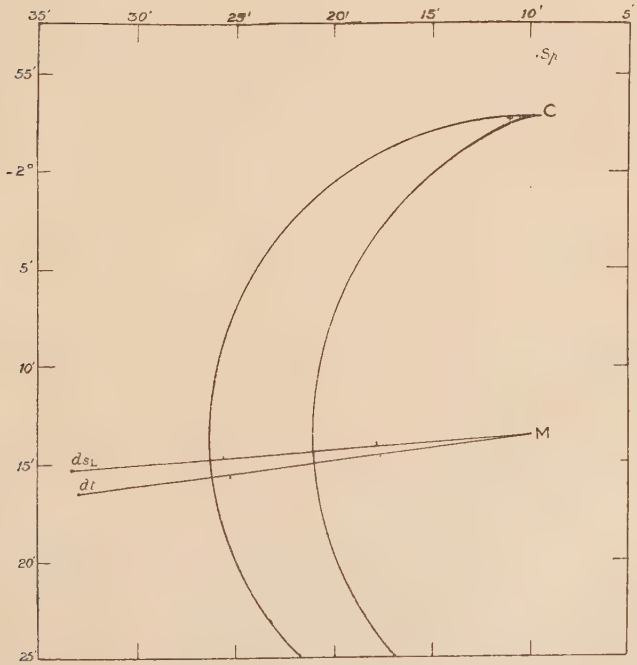
35′ 30′ 25′ 20′ 15′ 10′ 5′
·S/
55′
C
-2°
5′
10′
M
ds_L
15′
dt
20′
25′

FIG. 4.

for the correction in the Moon's longitude the difference between the longitudes of the star and of the cusp, or $+3303''$.

(ii.) In this case the time is that of the beginning of the observation. At Dr. Pickering's suggestion I have allowed a margin of $3'$ for a bright occultation, and I accordingly make the correction in longitude $+1640''$.

(iii.) The record seems to imply that the three eastern stars of the Pleiades were covered. A correction should be applied sufficient to cover 27 and 28 Tauri, but not sufficient to uncover

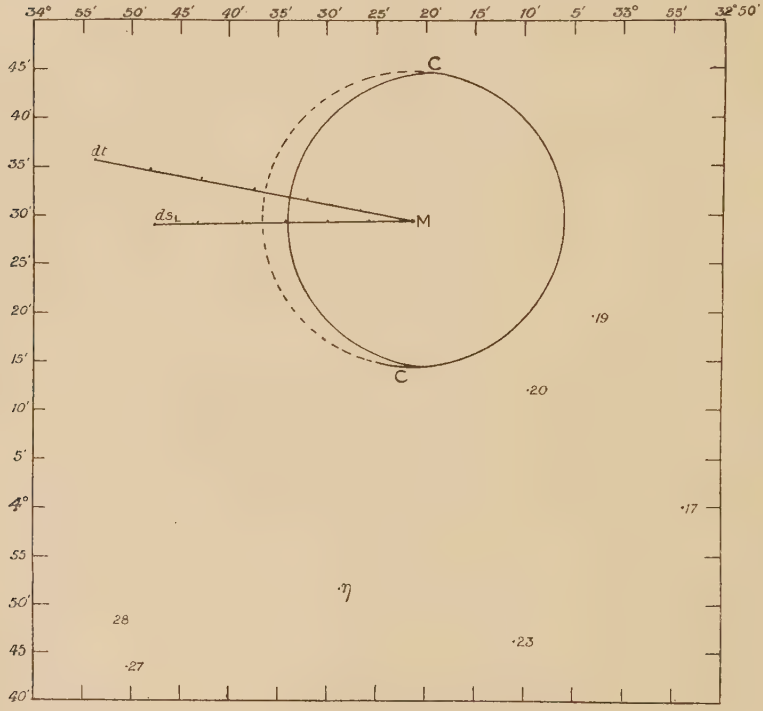

FIG. 5.

η Tauri. A correction of from $+360''$ to $+640''$ would satisfy these conditions, allowing as before $3'$ margin on the illuminated side of the Moon.

(iv.) Here there is no actual occultation. Spica is said to be touching the north limb. The computation makes its distance agree exactly with the $3'$ margin allowed by Dr. Pickering. As the longitude of the star exceeds that of the cusp by $+11''$, I assign that value to the resultant correction.

(v.) Here the Moon cannot be made to occult the south-east of the Pleiades. Agrippa must have intended the north-west of the Pleiades. A correction of from $-5'$ to $-23'$ appears to be required.

(vi.) The first stage of this observation will be satisfied if Spica is not more than 3′ east of the Moon's disk and not at all west of

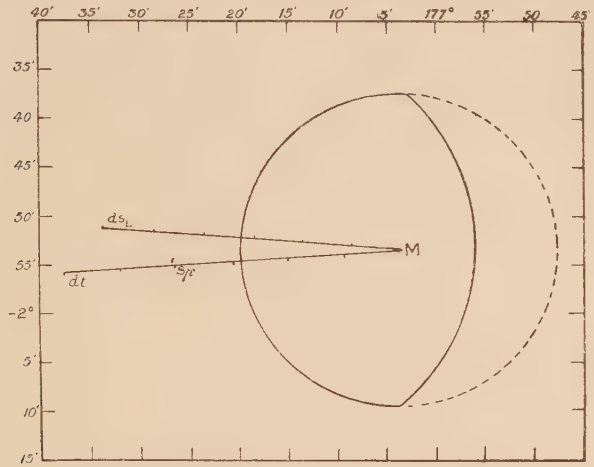

FIG. 6*a*.

it. Anything from +2′ to +38′ will satisfy this condition. In the second stage the distance from the Moon's centre must be at least a semi-diameter, because the star is visible, and it apparently

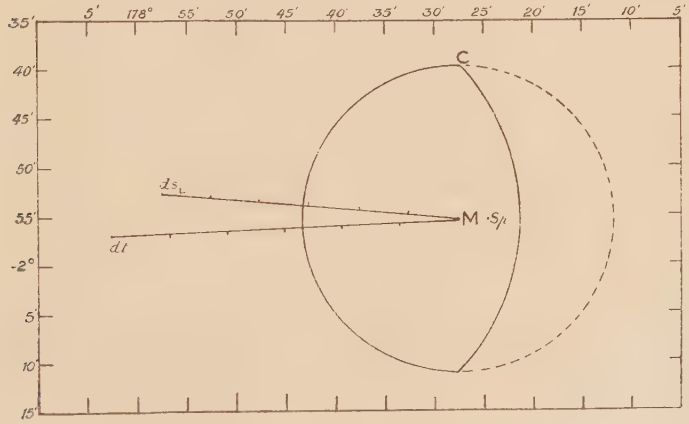

FIG. 6*b*.

approximates to, but falls perceptibly short of, the Moon's diameter. If we assume that the real distance was $\frac{4}{5}$ths of the Moon's diameter, a correction of +23′ to the longitude will be necessary, with a margin of about 5′ each way.

(vii.) The definite statement about the position of the Moon and her south cusp in relation to the line joining π Scorpii and δ Scorpii is inconsistent with Menelaus's inference that β Scorpii was in occultation. I find that the line joining π Scorpii and δ Scorpii would intersect the Moon's parallel of latitude in longitude $215^\circ\ 47'\ 34''$, making an angle of $83^\circ\ 53'\ 16''$ with the parallel. Refraction would reduce the apparent longitude both of the Moon and of this line by $74''$. A correction of $+519''$ to the longitude

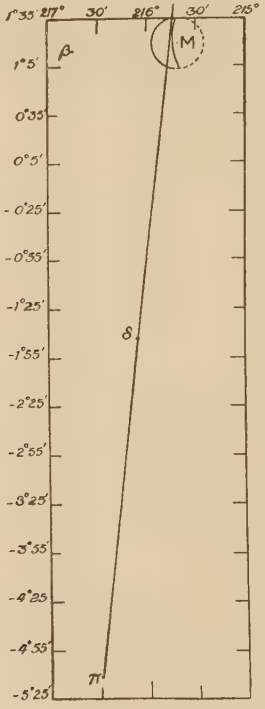

FIG. 7*a*.

would place the south cusp on the line joining the two stars, and would also place the centre of the Moon east of the line. I have therefore treated this as the correction derived from the observation.

The next problem is the weighting of the different observations. There would appear to be three possible sources of error: (1) vagueness of indication of time, (2) vagueness of indication of phase, (3) clock error. The third of these can only be determined from the discordances, but the first two can be readily allowed for. I assume from cause (1) a probable error in seasonal time of ± 5 minutes for most observations, but of $\pm 2{\cdot}5$ minutes for

observations I and VI*a*, where the time is given more precisely, and from cause (2) a probable error in longitude of one-fourth the

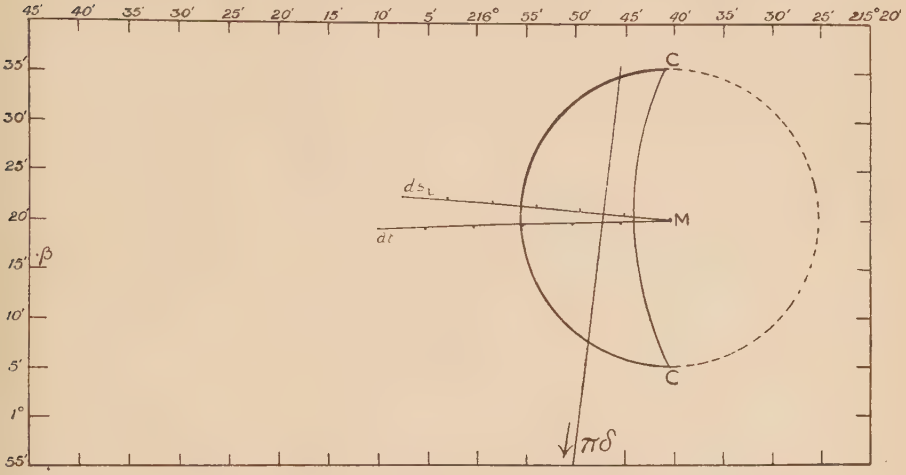

FIG. 7*b*.

range of longitude which will satisfy the observation. Clock error will be disregarded for the present. I thus tabulate—

No. of Observation.	Resultant Correction to Secular Acceleration.	Probable Error due to Cause (3).
I	$+\frac{3303}{398} = +8''\cdot30$	$\pm\frac{385}{398}\times\frac{2\cdot5}{14\cdot4}\times\frac{208\cdot72}{180} = \pm0''\cdot195$
II	$+\frac{1640}{406} = +4\cdot04$	$\frac{367}{406}\times\frac{5}{14\cdot4}\times\frac{185\cdot60}{180}$ 0·324
III	$+\frac{500}{439} = +1\ 14$	$\frac{355}{439}\times\frac{5}{14\cdot4}\times\frac{202\cdot02}{180}$ 0·315
IV	$+\frac{11}{468} = +0\cdot02$	$\frac{459}{468}\times\frac{5}{14\cdot4}\times\frac{197\cdot78}{180}$ 0·374
V	$-\frac{840}{262} = -3\cdot21$	$\frac{322}{262}\times\frac{5}{14\cdot4}\times\frac{217\cdot59}{180}$ 0·516
VI*a*	$+\frac{1200}{298} = +4\cdot03$	$\frac{340}{298}\times\frac{2\cdot5}{14\cdot4}\times\frac{220\cdot53}{180}$ 0·243
VI*b*	$+\frac{1380}{298} = +4\cdot63$	$\frac{349}{298}\times\frac{5}{14\cdot4}\times\frac{220\cdot53}{180}$ 0·498
VII	$+\frac{519}{270} = +1\cdot92$	$\frac{296}{270}\times\frac{5}{14\cdot4}\times\frac{219\cdot46}{180}$ 0·464

No. of Observation.	Probable Error due to Cause (2).	Probable Error due to Causes (1) and (2).	Weight.
	″ ″	″	
I	0 = 0	±0·195	13
II	0 = 0	0·324	5
III	$\pm\frac{70}{439} = \pm 0{\cdot}16$	0·353	4
IV	0 0	0·374	4
V	$\pm\frac{270}{262}$ 1·03	1·152	0
VI*a*	$\pm\frac{540}{298}$ 1·81	1·829	0
VI*b*	$\pm\frac{150}{298}$ 0·50	0·708	1
VII	0 0	0·464	2

This gives a mean correction to secular acceleration of +4″·9. The probable error resulting from the individual probable errors should be ±0″·13, while the probable error resulting from the discordances is ±1″·00. It is clear, therefore, that the clock errors greatly outweigh the errors which can be estimated in advance.

As the times are expressed in seasonal hours, it may be inferred that the time was discovered in each case by a water-clock, the ancient water-clocks being constructed so as to indicate such hours. It will be noticed that the observations were all made either in the evening not later than $8^h\ 43^m$ local mean time, or in the morning not earlier $15^h\ 30^m$ local mean time; not one observation was within three hours and a quarter on either side of midnight. Now, as seasonal hours were reckoned from sunset to sunrise the inference is that the times of the evening observations were compared with sunset and the times of the morning observations with sunrise. The probable error in the observed time ought therefore to be in proportion to the interval separating it from sunset or sunrise as the case may be.

In seeking the probable error in the observed time it is necessary to combine the two stages of observation VI, since the errors in these stages are doubtless systematic. Weighting the two observations according to the errors due to causes (1) and (2), I get for Greenwich mean time of observation $17^h\ 9^m$, for correction to secular acceleration +4″·55, and for probable error due to causes (1) and (2) ±0″·660. I now tabulate the factors of probable errors due to cause (3) for each observation, with the resultant weights. The first coefficient in each factor is the interval of time in degrees between the observation and sunset or sunrise.

No. of Obs.	Factor.	Weight.	No. of Obs.	Factor.	Weight.
I	$52''\cdot18 \times \frac{385}{398}$	4	V	$39''\cdot29 \times \frac{322}{262}$	4
II	$33''\cdot51 \times \frac{367}{406}$	11	VI	$23''\cdot44 \times \frac{348}{298}$	13
III	$47''\cdot70 \times \frac{355}{439}$	7	VII	$21''\cdot34 \times \frac{296}{270}$	18
IV	$41''\cdot20 \times \frac{459}{468}$	6			

This gives a correction to secular acceleration of $+2''\cdot6 \pm 0''\cdot70$.

If we had assumed that the probable error in observed time was equal at all hours of the night, we should have had $+2''\cdot6 \pm 0''\cdot90$, while on the improbable assumption that the error varied in each case with the interval elapsed since sunset we should have $+1''\cdot8 \pm 0''\cdot78$.

If the total probable error is $\pm 0''\cdot70$, and the probable error apart from the clock errors is $\pm 0''\cdot13$, the probable error due to the clock errors should be $\pm 0''\cdot69$. From which it follows that the probable error from this cause for weight 1 is $\pm 5''\cdot5$, equivalent to a probable clock error of about 12 minutes an hour. Gathering up the errors from all causes, we have,

No. of Observation.	Probable Error from Causes (1) and (2).	Probable Error from Cause (3).	Total Probable Error.	Weight.
I	$\pm 0''\cdot195$	$\pm 2''\cdot757$	$\pm 2''\cdot763$	4
II	0·324	1·654	1·685	11
III	0·353	2·106	2·135	7
IV	0·374	2·207	2·238	6
V	1·152	2·637	2·878	4
VI	0·660	1·512	1·650	11
VII	0·464	1·277	1·359	16

This confirms the result previously obtained, both as to the amount of the correction and the probable error. Adding the correction to the assumed acceleration, we get as the total sidereal secular acceleration of the Moon's mean motion,

$$+10''\cdot6 \pm 0''\cdot70 \text{ per century.}$$

If we deduct from this the theoretical acceleration of $+6''\cdot1$, we get an empirical acceleration of $+4''\cdot5 \pm 0''\cdot70$.

To this value we shall have to add a correction of $+0''\cdot05$ for each second that it may be found necessary to add to Dr. Cowell's value for the secular acceleration at epoch used in this paper. Any such correction would be far less than the probable error.

It must also be realised that the value obtained in this paper for the empirical acceleration is the mean value for the past twenty centuries. We have no means of knowing whether this value is

subject to secular change, and the range of the observations is not sufficient to enable us to detect such a change. It will also be observed that the value is greatly in excess of that determined from the lunar eclipses; and the difference can only be explained by a secular acceleration of the Sun's mean motion. It is interesting to observe that the lunar acceleration just obtained is almost identical with that obtained by Dr. Cowell in 1905, when he found from the solar eclipses a lunar acceleration of $+10''\cdot9$ and a solar acceleration of $+4''\cdot1$. Dr. Cowell's result was based on the assumption that a secular acceleration of the node is impossible; but it would appear that a small acceleration of the node is indicated by the magnitudes of the ancient lunar eclipses, which seem to give a correction to the elongation of the Sun from the node different from the acceleration of the Sun resulting from a comparison of the ancient occultations with the times of the ancient lunar eclipses. It seemed to me impossible to determine the position of the node from the observations discussed in this paper.

The secular acceleration of the Moon found in this paper ought to be compared with the result of the five astrolabe observations given in the Almagest, comparing the Moon with stars, but I did not consider it desirable to delay the publication of my work on the occultations till I could find time to discuss the astrolabe observations. A re-discussion of the ancient lunar eclipses might introduce slight modifications into some of the terms, while a discussion of the ancient planetary observations might not only provide independent evidence for determining the acceleration of the Sun, but might possibly determine similar accelerations to the planetary motions.

In his paper on the longitude of the Moon from 1750 to 1910 (*M.N.*, lxxiii. (1913), p. 711), Dr. Brown raises the question whether in tables of the Moon a purely theoretical value should be adopted for the secular acceleration or whether an observed value should be deduced from a combination of the ancient eclipses with modern observations. It appears to me that if empirical periodic terms are to be introduced at all, they should be adjusted so as merely to represent the unexplained residuum after the introduction of the best determined empirical secular terms, and should not be made to include the difference between the empirical and theoretical values of the secular terms. This must simplify the work of those who wish to investigate the empirical terms in the light either of ancient or of modern observations, and could just as accurately represent the future places of the Moon during the time in which the tables are likely to be in use.

6 *Blackhall Road, Oxford:*
1915 *March* 8.

Note on some Results of the New Determination of the Secular Acceleration of the Moon's Mean Motion. By J. K. Fotheringham, M.A., D.Litt.

I throw these results into a separate note, because I do not wish to mingle rough work with work of a more accurate character. In the preceding paper I have obtained + 10″·6 as the value of the sidereal secular acceleration of the Moon's mean motion at epoch 1800 Jan. 0·0, assuming Dr. Cowell's value for the centennial motion at that epoch. But the introduction of the revised value for secular acceleration into the discussion of the modern observations involves an addition of about 4″ to Dr. Cowell's value for the centennial motion at epoch. This will involve an addition of about 0″·2 to the secular acceleration as deduced from the ancient observations. I therefore propose to adopt + 10″·80 as the value to be introduced into the modern observations. Introducing Dr. Brown's corrections to the planetary terms given in *M.N.*, lxxiii. (1913) p. 701, and converting Newcomb's empirical term of long period by the formula given on p. 708 of the same paper, I obtain for the empirical term a value of $+ 15''\cdot43 \sin (1^\circ\cdot23t + 99^\circ\cdot6)$, where t is the time reckoned in Julian years from 1800 Jan. 0·0. This term should have a period of 292 years. The corrected expression for the mean longitude then becomes

$$335^\circ 43' 23''\cdot13 + 1732564392''\cdot92\mathrm{T} + 11''\cdot91\mathrm{T}^2 + 0''\cdot0077\mathrm{T}^3,$$

where T represents the number of Julian centuries elapsed since 1800 Jan. 0·0, the correction to Dr. Cowell's value being

$$-1'' + 4''\mathrm{T} - 0''\cdot1\mathrm{T}^2 - 0''\cdot005\mathrm{T}^3.$$

In his *Researches on the Motion of the Moon*, part ii. p. 208, Newcomb gives equations of condition for groups of ancient and mediæval lunar eclipses and modern occultations. I reject the Babylonian eclipses for two reasons, one of which was stated in my paper. (1) Because the times of observation have been very loosely reduced by Hipparchus; for the seventh, eighth, and ninth of the series the recorded times have given rise to such difficulties that it has been suggested that the eclipses were not really observed at Babylon. In the one case, the fifth on the list, where we have the cuneiform record, the time quoted by Ptolemy from Hipparchus is 38 minutes later than the time on the cuneiform tablet. The tablet makes it $1\frac{2}{3}$ double hours after nightfall, *i.e.* after sunset. As the sun set at $7^h\ 2^m$ local solar time, the observed time should be $10^h\ 22^m$ local solar time. Hipparchus renders "one hour before midnight."* (2) Because for comparison with the results

* It is very possible that Hipparchus added these 38 minutes to obtain the time of the middle of the eclipse from the observed time of the beginning. If so, he added about half as much as he should have done. This hypothesis only confirms the suggestion that these eclipses are not direct records of observations.

of the occultations it is desirable to have the eclipses spread over the same range of time as the occultations, so as to eliminate the effects of any fluctuation in the elements introduced into the computation.

For this purpose it is immaterial what lunar constants we use, provided the same are used for eclipses and occultations. Newcomb gives equations of condition for eight lunar eclipses with dates ranging from -200 to $+136$. Substituting the elements of the longitude found in this note to correspond to a secular acceleration of $+10''\cdot8$, and taking the probable errors from Newcomb, I deduce from these eclipses a secular acceleration of $+7''\cdot6\pm0''\cdot44$. This, of course, represents the acceleration of the Moon's motion as compared with the Sun, not the acceleration of her motion as compared with the stars. By deducting this from $+10''\cdot8\pm0''\cdot70$ we get $+3''\cdot2\pm0''\cdot83$ for the secular acceleration of the Sun. This difference cannot possibly be due to any other cause, as all other secular terms should be common to the occultations and the eclipses. Dr. Cowell, assuming that no correction was to be made to the node of the Moon's orbit, obtained from the solar eclipses $+4''\cdot1$ for the solar acceleration.

By a least squares analysis of the corrections to Dr. Cowell's acceleration of the Sun's distance from the node of the Moon's orbit as given in my paper (*M.N.*, lxix. (1909) p. 668), I find for the empirical correction to the acceleration of the Sun measured from the node, $+1''\cdot9\pm0''\cdot45$. Combining this with the value just obtained for the secular acceleration of the Sun, we get for the empirical correction to the secular acceleration of the Moon's node $+1''\cdot3\pm0''\cdot94$. If we substitute Dr. Brown's latest determination of the centennial motion of the node (*M.N.*, lxxiv. (1914) p. 562) for Dr. Cowell's value used in my paper, it will be necessary to make a corresponding reduction of $0''\cdot3$ in the secular acceleration of the node. The empirical correction should also be increased by $0''\cdot1$ to represent the difference between the values used by Dr. Cowell and Dr. Brown for the theoretical acceleration.

These corrections reduce the empirical correction to the acceleration of the node to $+1''\cdot1\pm0''\cdot94$. In view of the large probable error it may be doubted whether any such term exists.

In a paper read before the British Association in 1914, Dr. Brown compared the fluctuations in the Sun's motion with the fluctuations in the Moon's motion. It is possible that a similar relation holds between the accelerations. If so, the explanation of the empirical element in the lunar acceleration must lie in some cause which affects the apparent motion of the Earth by about $\frac{2}{3}$ as much as the apparent motion of the Moon.

6 *Blackhall Road, Oxford:*
1915 *March* 12.

Note on Dr. Fotheringham's Paper on the Occultations in the "Almagest." By E. B. Knobel.

Dr. Fotheringham states that he is not aware that the ancient occultations have ever been discussed.

Probably the earliest discussion of one of the occultations recorded in the *Almagest* is that by Nicholas Greenwood, in *Astronomia Anglicana*, 1689, which is as follows, verbatim :—

"In the year of Nabonassar 455 (*sic*) the 5th day of the month Tibi, at eight hours in the evening Tymocharis at Alexandria observed the easterly limb of the Moon to cover the Virgin's spike. This was Anno ante Christi 294, March the 9th Day, $5^h\ 49^m$ P.M. at London, at which time the Sun was by our Tables in ♓ 14° 32′ 48″.

	s	°	′	″
"Anom. med. ☾.	10	21	14	49
Equation add.		3	8	44
Anom. vera.	10	24	23	33
Apog. add.	6	26	26	50
☾ 1 equated.	5	20	50	23
Sol ver. Loc.	11	14	32	48
☾ à Sol.	6	6	17	35
Æquat 2d sub.	0	0	3	23
☾ in orb.	5	20	47	0
☊ sub.	6	11	27	16
Argum. Lat.	11	9	19	44
Reduct add.			4	37
☾ in Eclipt.	5	20	51	37
Lat. M. D.		1	56	54

		°	′	″
"Right Ascension of Med. Cœli.		105	48	0
Altitude of the Moon.		22	29	3
Dist. Moon from the vertex.		67	30	57
Parallax ☾ in altitude.		0	52	30
,, in Long. in ortum.		0	49	30
,, in Lat. Austr.		0	13	52
The visible center of the Moon.	♍	21	41	7
With Lat.		2	0	45
Semidiameter of the Moon add.			15	28
East limb of the Moon.	♍	21	56	35
The place of the star.	♍	21	42	59
Lat. of the star south.		1	59	0"

The error in the year of Nabonassar is probably a printer's error, as the year B.C. is correct.

In the translation of the *Almagest* into German by Dr. Karl Manitius (1913), he fully discusses all the occultations which have been considered by Dr. Fotheringham. His figures in some cases do not agree with those of the latter. Using Dr. Fotheringham's

numbers for the eclipses, Manitius gives the dates of I, IV, VI, and VII as December 21, November 9, January 11, and January 14 respectively, with, of course, corresponding differences in the Moon's longitude. The latitudes of all do not agree with those determined by Dr. Fotheringham.

I am not aware of any other investigation of these occultations.

1915 *March* 15.

Note by Dr. Fotheringham.

Dr. Manitius translates Ptolemy's discussion of the occultations with explanatory notes, but without reference to modern astronomy. His dates for the morning observations differ from mine in appearance, because he reckons the day from midnight, while I have reckoned it from noon. The differences in longitude are due partly to the defective value of secular acceleration, which I had adopted from Newcomb; partly to Ptolemy's error as to the mean motion of the Moon in relation to the equinox, to which I have referred in my paper; and partly to Ptolemy's ignorance of all but the largest inequalities in the Moon's longitude. I have not thought it necessary to investigate the differences in latitude.

J. K. Fotheringham.

1915 *March* 28.

Baxendell's Observations of Variable Stars. Fourth Instalment. No. 10, S Geminorum; No. 11, R Herculis; No. 12, S Herculis. Edited by H. H. Turner and Mary A. Blagg.

No. 10, *S Geminorum* (2742 *Chandler*).

$7^h\ 34^m\ 20^s\ +23^\circ\ 47'{\cdot}2$ (1855·0).
7 37 3 +23 41 ·1 (1900·0).

Identification of Comparison Stars.—In this case the identification of the comparison stars proved very difficult. Baxendell made no fewer than 21 diagrams on different dates, and these diagrams vary greatly in scale, in orientation, and in the relative positions of the stars, being evidently in most cases jotted down roughly and hastily without any attempt at exactitude. The designations for the comparison stars also vary greatly, some of these having as many as four different letters used for them at different times; and the same letter is also used for different stars on various occasions, four different stars being called "*b*" on different diagrams. The stars in a few diagrams were at first identified otherwise than as in Table I., but the resulting magnitudes were so very much too bright that the identifications were revised. It is still possible that an occasional mistake has been made. Though the letters "*a*" and "*b*" were reversed in the diagrams of 1868, yet they seem to have been used in the observation of April 11 in the usual manner, as "*a*" is made brighter than "*b*."

TABLE I.

Baxendell's Comparison Stars.

Hagen. No.	Hagen. Step.	H.A. Mag.	H.A. Let.	Baxendell's Diagrams. 1857. Nov. 18.	1858. Feb. 26.	1859. May 22.	1860. Mar. 15.	1860. Mar. 25.	1861. Oct. 28.	1861. Dec. 25.	1862. Nov. 16.	1863. Apr. 13.	1863. May 17.	1864. Mar. 23.	1864. May 7.	1864. Dec. 21.	1865. Nov. 21.	1866. Oct. 9.	1868. Apr. 11.	1868. Apr. 23.	1869. Feb. 5.	1869. Mar. 2.	1881. Dec. 12.	1885. Mar. 22.	Adopted Mag.
2	0	7·53	*a*	*b*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	16	8·45	*c*	*d*	...	...	*a*	...	*a*	...	...	...	...	*a*	...	...	*a*	*a*	*b*	*b*	*a*	...	A	*a*	86
8	20	(8·70)	...	*c*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	25	9·00	*d*	*e*	...	*n*	*b*	*a*	*b*	...	...	...	...	*b*	...	*a*	*b*	*b*	*a*	*a*	*b*	*a*	*a*	*b*	90
15	46	10·11	*f*	*f*	...	*x*	...	*x*	*c*	*x*	*x*	*x*	*x*	...	?	*b*	*c*	...	...	*x*	...	*b*	*b*	...	101
28	68	11·46	*h*	...	*w*	...	...	*y*	...	...	*y*	...	...	...	...	*c*	...	...	...	...	...	...	...	...	115
40	82	12·36	*l*	...	*x*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	124
62	122	13·61	*q*	...	*y*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	*c*	...	136

The only designation besides S given on the diagram of 1864 May 7 is a query against the star usually called "*x*," and the query is used in the observation to designate it. In 1861 Feb. to May, the designations *a*, *b*, and *c* are used as in the diagram of Oct. 28 of that year, and the stars 124 and 136 are called respectively "* S following" and "* n.p." The former is also sometimes called "companion," or "star near," in the years 1878 to 1884. The stars "*b*" and "*c*" of 1857 Nov. 18 are never used for comparison with the variable. The heading "H A." refers to *Harvard Ann.*, lvii., part 2, p. 238. The magnitudes adopted are those of Harvard, except in the case of one star, for which a slightly altered magnitude suits Baxendell's comparisons much better.

In Table II. Baxendell's differences between his comparison stars are shown. Cases where the number of comparisons does not exceed three have been omitted. The unit of comparison is Baxendell's "step."

TABLE II.

Baxendell's Differences between his Comparison Stars.

Years.	90–86.	101–90.	115–101.	124–115.	124–101.
1857–9	$4\cdot0_1$	$14\cdot2_2$	...	$7\cdot0_3$	
1860–1	$2\cdot8_9$	$14\cdot4_7$	...	...	35·0
1864–6	$4\cdot8_2$	$16\cdot0_3$	$8\cdot0_1$	...	$20\cdot0_1$
1868–9	$3\cdot0_1$	$15\cdot0_1$	...	...	...
1878	...	...	$6\cdot0_3$	...	...
1881	$5\cdot5_1$	$10\cdot0_1$	$10\cdot0_1$	...	..
1885–6	$3\cdot5_1$	...	$7\cdot0_1$	...	...
Means	3·4	14·5	7·2	7·0	30·0
No. of Obs.	15	14	6	3	3
Reduced Means	3·0	11·6	6·1	6·0	22·0
Adopted above	4·0	11·0	14·0	9·0	23·0

The reduced means are obtained by applying to each individual comparison the formula $t = 0\cdot915s - 0\cdot0115s^2$, where s is the value in steps and t in tenths of magnitude. This formula is found to represent the relation between s and t very accurately on the average in the whole series of observations of these variables, but there are certain exceptions in individual cases. In Table II. it will be seen that the reduced mean for 115 – 101 does not agree well with the Harvard value. But if we take the difference between Baxendell's values for 124 – 115 and 124 – 101 instead, we get (as reduced mean) 115 – 101 = 16, which is in good agreement for so large an interval. Except in the first two columns, the comparisons are too few in number to be of much importance, however.

In Table III. are given the observations, in which the formula of reduction given above has been applied throughout to the nearest tenth of a magnitude. The telescopes used were A 68 on 1859 May 11; 1860, from April 10; 1861 Jan. 4 to 28, Feb. 10 and 24, Mar. 16 to April 10, May 4, and from Oct. 28 to 1862 Jan. 22; 1862 Nov. 23 and Dec. 26; and 1863 Mar. 21 to 1869 Mar. 2, except on 1864 Oct. 7 and Nov. 5, when R (13) was used. The latter was used also 1858 Feb. 26 and 28 and Mar. 6, and (with power 196) Mar. 16; also from April 1 to May 6 with power 81, and on April 17 with power 199 also; 1859 Nov. 30 (p. 81 and 96); 1861 Mar. 13, April 14 and May 9 (p. 81); 1862 Nov. 16, and 1863 Feb. 8 (p. 81). A 40 was used 1858 Mar. 25, and 1859 May 13 to 29; A 223 on 1860 Mar.

21 and 22; 22 (*a*) 39 on 1860 Mar. 15 and 18 and April 1; T 35 on 1857 Nov. 18 and 23, and 1858 Jan. 9. E 80 was used apparently from 1877 onwards. C 39 was used on 1861 Jan. 1 and 3 (1st observation), Feb. 6 and Mar. 8; R (7) 39 on 1861 Jan. 3 (2nd observation); and R (7) 130 on same date (3rd observation). For full descriptions of these telescopes see *M.N.*, lxvii. p. 316, and lxxiii. 2, p. 126. Although the comparisons in the observations are given usually to the nearest tenth of magnitude only, half-tenths are given in the second observation of 1861 Jan. 3, to show the difference caused by the use of a different power and instrument.

TABLE III.

Baxendell's Observations of S Geminorum.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857			
Nov. 18	9637	90 − 3	87
23	9642	90 + 4, 101 − 1	97
1858			
Jan. 9	9689	Suspected.	
Feb. 26	9737	Invisible.	
Mar. 6	9745	,,	
16	9755	,,	
25	9764	,,	
Apr. 1	9771	,,	
4	9774	,,	
6	9776	,,	
10	9780	,,	
17	9787	,,	
May 3	9803	,,	
6	9806	,,	
1859	240+		
May 11	0176	101 − 2	99
13	0178	Barely glimpsed (twilight and strong moon.)	
16	0181	Invisible (twilight).	
20	0185	Not seen.	
22	0187	90 + 6, 101 − 11	93
23	0188	90 + 9	99
1859			
May 29	0194	Invisible, owing to trees and twilight.	
Nov. 30	0379	About 135.	
1860			
Mar. 15	0485	90 − 2	88
18	0488	90 + 1	91
21	0491	86 + 1, 90 + 0	88
22	0492	90 − 2	88
25	0495	86 + 1, 90 − 1	88
Apr. 1	0502	<90+0 (Hazy and moon).	
10	0511	90 + 4, 101 − 8	94
15	0516	90 + 4, 101 − 6	94
20	0521	90 + 9, 101 − 1	100
23	0524	90 + 9, 101 − 3	98
24	0525	90 + 9, 101 − 3	98
Oct. 11	0695	=130	
1861			
Jan. 1	0777	86 + 2, 90 − 3	88
3	0779	86 + 3, 90 − 1	89
3	0779	86+1·5, 90−0·5	88
3	0779	86 + 1, 90 + 0	88
4	0780	86 + 1, 90 − 2	88
6	0782	86 + 1, 90 − 1	88
15	0791	86 + 3, 90 − 0	90

TABLE III.—*Baxendell's Observations of S Geminorum*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1861			
Jan. 28	0804	90 + 4, 101 − 4	96
Feb. 6	0813	90+11, 101 − 6	98
10	0817	101 − 4	97
24	0831	101 − 2	99
Mar. 8	0843	101 + 2	103
13	0848	101 − 2	99
16	0851	101 − 1	100
21	0856	Invisible (moon).	
30	0865	101+11, 124−14	111
Apr. 3	0869	101+14, 124−11	114
10	0876	124 − 2	122
14	0880	124 + 0	124
May 4	0900	<125	
9	0905	136 + 2 (137)	138
Oct. 28	1077	101 − 4	97
Dec. 25	1135	101 − 4	97
28	1138	101 − 3	98
1862			
Jan. 22	1163	Invisible <125	
Nov. 16	1461	101 + 3	104
23	1468	124 + 5	129
Dec. 26	1501	Invisible <120	
1863			
Feb. 8	1545	,, <140	
Mar. 21	1586	,, <130	
23	1588	,, <130	
Apr. 13	1609	Glimpsed=130	
23	1619	101 + 3	104
May 17	1643	101 − 3	98
Dec. 5	1845	Not seen <120	
1864			
Mar. 23	1954	86 + 3, 90 + 1	90
28	1957	90 + 2	92
May 7	1997	101 − 3	98
Oct. 7	2152	<135	
Nov. 5	2181	? Glimpsed <135	
Dec. 21	2227	101 + 0	101
30	2236	101 − 3	98
1865			
Jan. 4	2241	90 + 6, 101 − 6	96
6	2243	90 + 6, 101 − 6	96
20	2257	90+11, 101 − 4	99
Mar. 1	2297	101 + 8, 124 − 8, 115 + 2	114
30	2326	124 + 4	128
Nov. 21	2562	90 + 1	91
Dec. 14	2585	86 + 9, 101 − 2	97
22	2593	101 + 2	103
1866			
Mar. 13	2674	Glimpsed 135	
Oct. 9	2864	86 + 6, 90 + 0	91
1867			
Jan. 8	2975	Invisible <133	
11	2978	<133	
28	2995	Not seen <134	
Feb. 20	3018	,, <130	
1868			
Apr. 11	3434	86 + 2, 90 − 1	88
23	3446	101 − 3	98
1869			
Feb. 5	3734	90 + 4	94
Mar. 2	3759	90 + 9, 101 − 3	98
1870			
Mar. 25	4147	Invisible.	
1871			
Mar. 13	4500	<130	
1878			
Feb. 7	7023	101 + 5	106
20	7036	Glimpsed (hazy and moon).	
Mar. 27	7071	Not seen <120	

TABLE III.—*Baxendell's Observations of S Geminorum*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1878			
Apr. 5	7080	136 − 8	128
Nov. 4	7293	86 + 4, 101 − 4	94
25	7314	201 − 3	98
Dec. 5	7324	115 + 2	117
7	7326	115 + 3	118
23	7342	124 + 0	124
30	7349	124 − 6	118
1879			
Oct. 15	7638	About 130.	
20	7643	124 + 7	131
1880			
May 5	7841	101 − 3	98
1881			
Apr. 1	8172	101 − 2	99
Oct. 26	8380	124 + 3	127
Nov. 14	8399	101 + 7, 115 − 2	110
1881			
Nov. 17	8402	101 + 1	102
Dec. 12	8427	90 + 3, 101 − 5	94
27	8442	86 − 1, 90 − 6	84
1882			
Mar. 12	8517	124 − 3	121
1883			
Feb. 23	8865	< 120	
1884			
Dec. 20	9531	124 + 4	128
1885			
Mar. 22	9623	86 + 2, 90 − 1	88
27	9628	90 − 1	89
1886			
	241+		
Nov. 19	0230	101 + 0	101
Dec. 4	0245	101 + 3, 115 − 3	103

These observations have been dealt with in the same manner as in the case of the stars included in the preceding instalments of this series. Like some of these, S Geminorum was invisible in Baxendell's instruments when near its minimum, and fully one-third of the light curve is missing in consequence. The observations are also very scanty, and there is necessarily a gap of at least five months in every year owing to the position of the star, so that it is rather a surprise to find that the mean of all the observations in each of the eight "portions" available give a very smooth curve, as far as it goes. In portions 6 and 11, the two observations in each case are taken earlier than the central date of those portions, and they have been placed in their true position in forming the curve, instead of in the centre of the portion. Table IV. shows the results of comparing each observation with the corresponding point in the curve. The mean is given for each portion in each period, the subscript figure showing the number of observations. The length of period assumed was 294 days, Chandler's formula being $2397546 + 293{\cdot}8$ E. The observations being so irregularly scattered, it is not possible to attempt much in the way of discussion, but the means of periods V. to X. and of periods XXVI. to XXXI. have been taken, respectively, and corrections to epoch obtained from them, amounting to $-1{\cdot}7$ days for the first, and $+1{\cdot}4$ days for the second.

TABLE IV.

Comparison of the Observations with a Mean Light Curve.

Period.	Initial Date.	No. and Mean Magnitude of Portion. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
	239+	93.	96.	102.	111.	126.	(135.)				(132.)	114.	99.
I.	9600	...	-5_2	...	...	...	...	...	...	...	...	...	...
II.	9894	...	...	...	...	...	...	...	...	...	...	...	$-0^{\cdot}5_2$
	240+												
III.	0188	$+5_1$	...	...	...	...	...	...	...	...	...	..	...
IV.	0482	$-4^{\cdot}5_5$	$+1_5$	...	...	...	...	...	...	...	...	...	...
V.	0776	-5_7	$+1_3$	-2_3	-3_3	$+1_2$	$+5_1$	...	...	...	...	...	...
VI.	1070	$+4_1$	...	-6_2	...	...	...	...	...	...	...	...	...
VII.	1364	...	...	...	-4_1	$+6_1$	...	...	...	...	...	-11_1	-2_1
IX.	1952	-3_2	0_1	...	...	...	...	...	...	...	...	...	0_4
X.	2246	$+6_1$	...	$+15_1$	...	...	...	...	...	...	...	...	...
XI.	2540	-3_1	-1_1	$+3_1$	...	...	...	...	...	...	...	...	...
XII.	2834	...	-4_1	...	...	...	...	...	...	...	...	...	...
XIV.	3422	-6_1	$+4_1$	...	...	...	...	...	...	...	...	...	...
XV.	3716	$+1_1$	$+1_1$	...	...	...	...	...	...	...	...	...	...
XXVI.	6950	...	...	...	0_1	...	-5_1	...	...	...	...	...	...
XXVII.	7244	...	...	-6_2	$+8_2$	0_2	...	...	...	...	...	...	...
XXVIII.	7538	...	...	...	...	$+7_1$	...	...	...	...	...	...	...
XXIX.	7832	$+5_1$	...	...	..	...	...	...	...	...	..	...	...
XXX.	8126	...	$+1_1$	...	...	...	...	...	...	...	...	$+11_1$	$+4_2$
XXXI.	8420	-4_2	...	...	$+3_1$	...	..	...	...	...	...	...	...
XXXV.	9590	...	-7_2	...	...	...	...	...	...	...	...	...	...
	241+												
XXXVII.	0184	...	$+3_1$	$+1_1$	...	...	...	...	...	...	...	...	...
$p-f$		...	+3	−9	−14	−24	−24	...	...	...	...	...	+33	+21
(Mean) V.+(5)		...	−3	+1	−0·5	−3	+3	(+5)	...	...	...	...	−11	−0
,, XXVI.+(5)		...	−1	+1	−6	+5	+2	(−5)	...	...	...	...	+11	+4

The results are here given and compared with Chandler.

The place of maximum, as read from the curve, is about 10 days after the zero point.

Group.	Mean E.	Chandler's Epoch.	Baxendell's Epoch.	Bax.–Ch. (a).	Bax.–Ch. (b).
		239+	240+		
V.+(5)	14	7546+4113·2	1668−1·7	+ 7·1	+2·2
		=1659·2	=1666·3		
XXVI.+(5)	35	7546+10283	7842+1·4	+14·4	+2·2
		=7829	=7843·4		

The correction to period from Baxendell's observations is $+\frac{3\cdot1}{21}$ days $= +0^{d}\cdot15$ per period, which would make the period $294^{d}\cdot15$ instead of Chandler's $293^{d}\cdot8$. If we substitute this value, we get for Chandler's epochs 2401664·1 and 2407841·2, giving Bax.–Ch. (b) $= +2\cdot2$ days only, which is very close. The differences in (a) might, however, be due to a periodic variation in the length of the period, and not to an error in Chandler's figure.

The corrections to epoch obtained above are, of course, unworthy of much reliance, but the place of maximum in the curve is very fairly well defined. The curve is too incomplete for analysis.

No. 11, *R Herculis* (5770 *Chandler*).

$15^{h}\ 59^{m}\ 43^{s}\ +18^{\circ}\ 45'\cdot9$ (1855·0).
$16\ \ 1\ \ 44\ \ +18\ \ 38\cdot4$ (1900·0).

TABLE V.

Comparison Stars.

Baxendell.							A.S.V. (II.)		Harvard		Adopted Mag.
1859 May 13 (and earlier).	1862 Mar. 3.	1864 July 13.	1864 Aug. 21.	1865 June 9.	1866 May 15.	Large Chart	No.	Grade.	Letter.	Mag.	
v	...	...	*a*	*a*	*a*	*a*	3	11	*f*	8·83	88
f	...	...	*b*	...	*b*	*b*	4	17	*h*	9·04	91
w	...	...	*c*	*b*	*c*	*c*	5	22	...	(9·18)	92
d	...	...	*d*	*c*	...	*d*	7	31	*l*	9·40	94
1	...	1	...	...	...	*e*	12	58	...	(10·08)	100
2	*c*	...	*w*	...	...	*w*	15	63	*n*	10·23	103
3	...	...	...	...	...	*z*	18	80	*o*	10·85	108
...	...	4	*x*	...	...	*x*	22	97	*p*	11·52	115
? 4	...	...	...	*d*	...	...	10	51	...	(9·87)	98
6	...	...	...	...	...	...	?(13	58	*m*	10·08)	102
e	..	.	..	..	...	...	?	?	...	...	90

The Harvard magnitudes are from the table on p. 194 of *H.A.* 37. Those in brackets are obtained by interpolation as on p. 200. The last two in the list cannot be identified, and their magnitudes have been assumed in agreement with Baxendell's comparisons. It will be seen that Baxendell varied his symbols considerably. The star $e=1$ was often called "lowest of triangle" or "brightest of triangle," the star $c=w$ was once called "top star," and the star x was called z in 1865. The identification of "4" with the "*d*" of 1865 is very doubtful.

TABLE VI.

Baxendell's Differences between his Comparison Stars.

Year.	91–88.	92–91.	94–92.	92–88.	94–88.	90–88.	103–103.	94–91.	92–90.
1857–9	$3^{\cdot}1_{23}$	$0^{\cdot}5_{27}$	$2^{\cdot}2_{34}$	$3^{\cdot}6_{23}$	$6^{\cdot}0_{20}$	$2^{\cdot}7_{5}$	$3^{\cdot}5_{4}$	$3^{\cdot}2_{24}$	$1^{\cdot}7_{6}$
1860–2	$3^{\cdot}0_{6}$	$1^{\cdot}0_{1}$	...	$5^{\cdot}0_{1}$	...	...	$6^{\cdot}0_{2}$	$3^{\cdot}0_{1}$	...
1864–8	$4^{\cdot}8_{2}$	$2^{\cdot}7_{11}$	$2^{\cdot}1_{5}$	$6^{\cdot}6_{5}$	...	...	...	...	...
Mean	3·2	1·2	2·2	4·2	6·0	2·7	4·3	3·2	1·7
No. of Comparisons.	31	39	39	29	20	5	6	25	6
Reduced Means.	2·8	1·0	2·0	3·7	5·3	2·5	3·6	2·8	1·6

The reduced means are obtained by applying the usual formula of reduction to each separate interval. They agree very fairly well with the Harvard magnitudes, but there appears to be a gradual increase in Baxendell's estimated intervals in several cases.

The telescope used was A 68, with the following exceptions:—

On 1857 Aug. 16, 20, 24, 28; and 1858 Mar. 6, Aug. 5: 30 (*a*) was used.

1857 Sept. 22 to 1858 Feb. 17: T 35.

1858 July 1 to 17, 19, Aug. 10 to 1859 May 11, 21, 28, 29, June 8, 25, July 10, and throughout 1860: 22 (*a*).

1858 Mar. 16 to Apr. 20, June 13 to 21, and July 18: R (7) 40; May 1: R (7) 81.

1858 May 19; 1861 Apr. 13; and 1862 May 16: R (13) 81.

1859 May 12 to 20, 22 to 27, 31, June 7, 16, 22, 26, 30: A 40.

1861 Mar. 19 to Apr. 6, Apr. 16; 1862 July 19, 24; 1863 Feb. 14, May 8; 1868 Mar. 14: C 39.

1878 June 10: E 80.

TABLE VII.

Baxendell's Observations of R Herculis.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857				1857			
Aug. 16	9543	92 + 0	92	Sept. 7	9565	88 + 0, 92 − 3	88
20	9547	92 − 2	90	12	9570	88 − 1, 92 − 4	88
22	9549	88 + 2, 91 − 2	90	16	9574	88 − 1, 92 − 4	88
24	9551	92 − 3	89	19	9577	88 − 1, 92 − 3	88
25	9552	92 − 3,	90	22	9580	88 − 2, 92 − 3	88
		R − 88 = 2 × (91 − R)		25	9583	88 − 2, 92 − 3	88
26	9553	92 − 3	89	28	9586	88 − 2, 92 − 4	87
28	9555	92 − 3	89	Oct. 4	9592	91 + 0, 92 − 3	90

TABLE VII.—*Baxendell's Observations of R Herculis*—continued.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857			
Oct. 6	9594	91 + 0, 94 − 3	91
8	9596	88 + 1, 91 + 0, 92 − 2, 94 − 3	90
Nov. 6	9625	Suspected < 94 + 6	
1858			
Jan. 16	9696	Invisible.	
Feb. 17	9728	,,	
Mar. 6	9745	,,	
16	9755	,,	
26	9765	,,	
Apr. 9	9779	,,	
20	9790	,,	
May 1	9801	,,	
19	9819	,,	
June 13	9844	103 + 2, 108 − 3	105
17	9848	100 − 1	99
21	9852	100 − 3	97
July 1	9862	94 + 3	97
10	9871	90 + 0, 94 − 3	90
17	9878	90 + 0	90
18	9879	91 + 0, 100 − 9, 90 + 0	91
19	9880	91 + 0, 88 + 2, 92 − 2	90
30	9891	91 + 2, 94 − 1, 88 + 4, 92 + 0	92
Aug. 5	9897	Invis. ; at least 92 + 6	
6	9898	92 + 4, 94 + 3	96
10	9902	94 + 0, 100 − 5	94
12	9904	94 + 3,	97
Sept. 5	9928	Invisible.	
1859			
	240+		
Feb. 28	0104	,,	
Mar. 31	0135	Suspected.	
Apr. 6	0141	Invisible.	
9	0144	,,	
21	0156	Glimpsed.	

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1859			
May 1	0166	94 + 2, 100 − 1	98
2	0167	94 + 0	94
4	0169	94 + 0, ?98 + 0	?96 or 94
8	0173	94 − 1	93
11	0176	91 + 1, 92 + 0, 94 − 2	92
11	0176	92 + 0, 94 − 3	92
12	0177	91 + 0, 92 + 0, 94 − 3	91
13	0178	91 + 0, 92 + 0, 94 − 2	92
16	0181	91 − 1, 92 − 1, 94 − 4	90
20	0185	94 − 3, 88 + 2, 91 − 1, 92 − 1	90
21	0186	91 − 1, 92 − 1	90
22	0187	91 − 1, 92 − 1, 94 − 3	91
23	0188	91 − 1, 92 − 1, 94 − 3	91
25	0190	88 + 1, 91 − 2, 92 − 2, 94 − 4	90
27	0192	88 + 1, 91 − 3, 92 − 3, 94 − 5	89
28	0193	88 + 0, 91 − 3, 92 − 3, 94 − 6	88
29	0194	88 + 1, 91 − 2, 92 − 2, 94 − 6	89
31	0196	88 + 1, 91 − 3, 92 − 3, 94 − 6	88
June 7	0203	88 + 1, 91 − 3	88
8	0204	88 + 1, 91 − 2	89
10	0206	88 + 1, 91 − 2, 92 − 2, 94 − 4, 90 − 1	89
14	0210	88 + 2, 91 − 1, 92 − 2, 94 − 4	90

TABLE VII.—*Baxendell's Observations of R Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1859			
June 16	0212	88 + 2, 91 − 1, 92 − 1, 94 − 4	90
22	0218	88 + 3, 91 + 0, 92 − 1, 94 − 4	91
25	0221	91 + 3, 92 + 3	94
26	0222	91 + 1, 92 + 0, 94 − 3	92
30	0226	91 + 2, 92 + 1	93
July 10	0236	94 + 2	96
13	0239	94 + 2	96
1860			
Mar. 14	0484	94 + 2, ?98 + 0	?97 or 96
Apr. 5	0506	88 + 0	88
8	0509	88 + 0	88
8	0509	88 − 1	87
11	0512	88 + 0	88
16	0517	88 + 0	88
20	0521	88 + 0	88
23	0524	88 + 2, 91 − 1	90
24	0525	88 + 0	88
27	0528	88 + 1, 91 − 1	90
May 1	0532	88 + 3, 91 + 0	91
9	0540	91 + 0	91
13	0544	91 + 1	92
16	0547	91 + 2	93
1861			
Mar. 19	0854	88 − 3	85
31	0866	88 − 2	86
Apr. 6	0872	88 + 0	88
8	0874	91 + 0	91
13	0879	91 + 0	91
16	0882	91 + 2, 94 − 1	93
May 17	0913	102 + 2, 108 − 3	104
1862			
Mar. 3	1203	163 + 1	104
Apr. 8	1239	Not seen < 120	
29	1260	?Glimpsed < 133	
1862			
May 16	1277	Not seen < 135	
July 19	1341	Not seen < 100	
24	1346	,, < 100	
28	1350	?Glimpsed < 130	
Aug. 22	1375	Invisible < 133	
Sept. 1	1385	,, < 125	
10	1394	,, < 125	
15	1399	,, < 120	
20	1404	,, < 130	
Oct. 4	1418	,, < 120	
14	1428	,, < 125	
18	1432	,, < 125	
24	1438	108 + 0	108
29	1443	103 − 1, 108 − 6	102
Nov. 24	1469	91 + 2, 92 + 1	93
1863			
Mar. 16	1851	Invisible < 130	
Apr. 13	1609	,, < 110	
23	1619	,, < 132	
May 8	1634	,, < 105	
17	1643	,, < 133	
June 20	1677	,, < 125	
23	1680	,, < 125	
July 3	1690	,, < 125	
Aug. 24	1742	,, < 125	
Sept. 5	1754	,, < 130	
Nov. 2	1812	91 + 5, 100 − 2	97
1864			
Apr. 12	1974	Not seen < 125	
25	1987	,, < 130	
May 7	1999	,, < 132	
13	2005	,, < 125	
26	2018	,, < 132	
June 8	2031	,, < 120	
July 4	2057	?Glimpsed < 130	
13	2066	115 − 1	114
26	2079	100 − 1	99
Aug. 21	2105	88 + 1	89

TABLE VII.—*Baxendell's Observations of R Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1864			
Aug. 26	2110	88 − 1	87
Sept. 24	2139	91 + 0, 92 − 2	90
26	2141	91 + 1, 92 − 1	92
Oct. 3	2148	91 + 2, 92 − 1	92
1865			
Apr. 2	2329	Not seen : at least 115+3	
26	2353	Not seen < 133	
May 15	2372	Glimpsed 130	
24	2381	115 + 2	117
June 9	2397	94 + 2, 98 − 2	96
11	2399	92 + 1, 94 − 2	92
17	2405	92 − 2	90
20	2408	88 + 3, 92 − 3	90
July 1	2419	88 + 1, 92 − 3	89
13	2431	88 + 0	88
21	2439	88 + 2, 92 − 4	89
Aug. 3	2452	88 + 5, 92 − 1	92
25	2474	92 + 1, 94 + 0	94
Sept. 7	2487	100 − 1	99
28	2508	115 − 1	114
Oct. 7	2517	?115 + 0*	?115
1866			
Apr. 21	2713	115 − 6	109
22	2714	103 + 1, 115 − 7	106
23	2715	103 − 1, 115 − 9	104
24	2716	103 + 0	103
May 7	2729	92 + 0, 94 − 2	92
15	2737	91 + 1, 92 − 1	92
May 17	2739	91 + 1, 92 − 1	92
20	2742	91 + 0	91
22	2744	91 + 0	91
26	2748	91 + 0	91
June 8	2761	91 + 0, 92 − 2	90
25	2778	91 + 2, 92 + 0	92
July 11	2794	91 + 3, 92 + 0	93
16	2799	91 + 6, 92 + 3	96
Aug. 20	2834	Glimpsed ? 115 + 1	?116
31	2845	115 − 1	114
Sept. 17	2862	< 115 + 3	
1867			
Apr. 11	3068	88 − 6	82
21	3078	88 − 1	87
July 24	3172	Glimpsed 133	
1868			
Mar. 14	3406	91 + 0	91
Apr. 9	3432	91 + 3, 94 + 0, 92 + 2	94
11	3434	91 + 3, 92 + 0, 94 − 2	93
May 11	3464	103 + 4, 115 − 4	109
30	3483	Not visible < 125	
June 7	3491	,, < 125	
1869			
Mar. 6	3763	,, < 100	
Apr. 12	3800	130	
1878			
June 10	7146	Invisible < 120	

In this case, as in many others, Baxendell was unable to observe the star near its minimum, so that, when the observations were dealt with in the usual manner, there remained a gap of five complete "portions," while two other portions contain more instances of the observation "invisible" than of estimates of magnitude. Nevertheless, a fairly smooth curve has been formed for the seven "portions" available, and the individual observations have been compared with it in the usual manner. The maximum

* The entry is "R small star following triangle."

point in the curve is obviously in portion 3, but the exact point is difficult to fix owing to the flatness of the curve. It appears to be at about 72 days (more or less) after the zero point. The observations of portion 3 have been divided into two sub-portions, and those in several other portions have been placed in their true position, according to the date of the observation, instead of in the centre of the portion, in forming the curve. The period assumed was 318 days.

TABLE VIII.

Comparison of the Observations with a Mean Light Curve.

Period.	Initial Date. 239+	Number and Mean Magnitude of Portion. 1. (123).	2. 96.	3. 89.	4. 91.	5. 97.	6. 108.	7. (116).	8.	9.	10.	11.	12.
I.	9500	...	-2_5	-1_6	-1_6	...	...	...	...	...	...	...	...
II.	9818	...	-1_4	$+1_5$	$+6_3$	...	...	...	...	...	...	...	..
	240+												
III.	0136	...	-3_{13}	0_{10}	$+3_6$	...	...	...	...	...	..	...	..
IV.	0454	...	-5_2	0_8	$+1_3$	...	...	...	.	...	.	...	...
V.	0772	...	...	...	-4_4	-2_2	-2_1	...	...	...	...	...	...
VI.	1090	...	...	...	...	$+9_1$	...	...	...	...	...	...	...
VII.	1408	...	$+4_2$	$+4_1$	...	...	...	...	...	..	...	. .	...
VIII.	1726	...	...	...	$+7_1$	...	...	...	...	...	...	...	...
IX.	2044	$+1_1$	0_1	-1_2	0_3	...	...	..	...	...	...	...	...
X.	2362	-1_1	-4_4	-1_3	$+2_1$	-1_2	$+6_1$	...	...	...	...	...	...
XI.	2680	...	$+5_5$	$+2_5$	$+1_2$	-2_2	...	0_1	...	...	...	...	...
XII.	2998	...	...	-7_1	-2_1	...	...	...	...	...	...	...	...
XIII.	3316	...	...	...	$+1_1$	-3_2	$+1_1$	...	...	...	...	...	...
A.	I+(4)	...	−3	0	+1	−2	−2	...	...	...	...	...	...
B.	IX+(4)	0	+1	0	0	−2	+3	0	...	...	...	...	...
	p–f	...	+34	+5	−8	−17	−19	...	...	...	...	...	...

Chandler's revision (A) 5537 gives the formula:

$$2402440 + 317{\cdot}7E + 17 \sin(10^\circ E + 322^\circ).$$

Corrections to epoch from the above Table give, for A, −3·3 days; and for B, +0·8 days. The figures have been weighted by *p–f* and by the number of periods in which the portion was observed. The following is a comparison between these results and Chandler's formula:—

Group.	Mean E.	Chandler's Epoch. 240+	Baxendell's Epoch. 240+	Bax.–Ch. d.
I.+(5)	−7	2440 − 2223·9 − 16·2 = 0199·1	0136 + 72 − 3·3 = 0204·7	+5·6
IX.+(5)	+1	2440 + 317·7 − 7·7 = 2750·0	2680 + 72 + 0·8 = 2752·8	+2·8

If the maximum point in the Baxendell curve be taken 4 days earlier (as it might well be), the differences between Baxendell and Chandler become $+1^{d}\cdot6$ and $-1^{d}\cdot2$, which are very small.

It is, of course, impossible to analyse the curve, and the corrections to epoch, obtained above, are of very little value, owing to the scantiness of the observations from which they are derived.

No. 12, *S Herculis* (6044 *Chandler*).

$16^{h}\ 45^{m}\ 18^{s}\ +15^{\circ}\ 11'\cdot4$ (1855·0).
16 47 21 +15 6·6 (1900·0).

TABLE IX.

Comparison Stars.

Names generally used till 1863.	1860	Baxendell's Diagrams. 1863			1864	1865	1866	Charts.		A.S.V.		Harvard.		Adopted Mag.
								M.	..	No.	Grade.	Let.	Mag.	
	Apr. 23.	Apr. 23.	July 20.	Aug. 17.	Apr. 25.	Nov. 26.	July 16.							
49	49	...	...	...	...	49	49	*a*	*a*	1	...	*b*	6·41	64
...	...	...	...	...	...	...	...	....	*b'*	2	...	...	6·93	69
p	...	...	...	...	...	...	...	*b*	*b*	...	...	...	[7·55]	74
f	...	...	*f*	...	*a*	*c*	...	*c*	*c*	3	0	*f*	7·60	76
w	...	...	...	...	*b*	*c'*	...	*c'*	*c'*	4	6	*g*	8·09	81
e	*e*	...	*e*	...	*e*	...	...	*d*	*d*	5	10	*h*	8·34	83
x	*x*	...	*x*	*y*	...	...	...	*q e*	*e*	6	16	*k*	8·79	88
y	...	<	...	...	...	...	*f*	*p* or *f*	*f*	7	17	...	(8·86)	89
"Unnamed near S."	...	...	...	...	...	...	...	*g*	*g*	11	40	...	(10·26)	96
"Unnamed near 49."	...	...	...	*z*	...	...	...	*h*	*h'*	10	40	*p*	10·26	103
...	...	...	...	...	...	...	...	*x*	*x*	13	49	...	(10·64)	106
...	...	...	...	...	...	...	...	...	*z*	29	112	...	(12·00)	...
...	...	...	...	...	...	...	...	*y*	*y*	31	120	...	(12·12)	121

The Harvard magnitudes are from the Table on p. 195 of *H.A.* 37; those in round brackets having been interpolated from that on p. 200. That in square brackets is obtained by interpolation from measures on a Franklin-Adams Chart, using Harvard data. The identification of the comparison stars in the earlier years proved difficult. A chart is alluded to in 1857, which has not been found, but there are five hastily made diagrams in the years 1860 to 1864, which (as shown above) are not all in agreement with each other. They do, however, supply a clue, and a further clue is found in the fact that the intervals between the stars usually then called *p*, *f*, *w*, *e*, *x*, *y*, and "unnamed near S," are sensibly consistent respectively with those between the stars called *b*, *c*, *c'*, *d*, *e*, *f*, and *g*, in later years. On 1863 May 17 and 22, the comparison stars used are called "*a*" and "*b*," but they are almost certainly the same as those used on May 8. On 1868 April 11, there is a note "Chart M," but the nomenclature of that

chart was first used in 1865. Baxendell's intervals agree fairly well with the Harvard magnitudes, except in the case of the star "*y*," which he evidently "saw" much brighter than 10·26. It is perhaps doubtful if his "*y*" was really that of the charts. The magnitudes obtained from comparisons with "*a*" and "*b'*" give results very much brighter than the corresponding magnitudes in other periods, but it is, of course, possible that the variable was really abnormally bright on those occasions. Both "*x*" and "*y*" are generally called "*x*" in the later series of observations, while in the earlier series *both* the anonymous stars are sometimes spoken of simply as "unnamed."

TABLE X.

Baxendell's Differences between his Comparison Stars.

Years.	69–64.	76–74.	81–76.	83–81.	88–83.	89–88.	96–89.	103–96.	121–103.
1857–60	...	$2\cdot6_8$	$4\cdot4_{22}$	$1\cdot5_{20}$	$3\cdot2_{14}$	$0\cdot9_9$	$10\cdot2_3$	...	...
1862–63	...	$3\cdot7_5$	$5\cdot5_{11}$	$1\cdot0_1$	$5\cdot8_5$	$2\cdot8_2$	...	...	...
1864–68	...	$2\cdot0_6$	$5\cdot4_5$	$0\cdot8_2$	$3\cdot8_5$	$3\cdot3_3$	...	...	...
1878–82	...	...	...	...	$5\cdot0_4$	...	$6\cdot0_6$	$10\cdot1_5$	$21\cdot0_1$
1883–86	$5\cdot2_2$	...	$5\cdot3_9$	$1\cdot0_4$	$3\cdot5_2$	$1\cdot5_1$	$8\cdot6_4$	$7\cdot0_2$	$27\cdot5_1$
Mean	5·2	2·7	4·9	1·4	4·0	1·7	7·8	9·2	24·2
No. of Comparisons.	2	19	47	27	30	15	13	7	2
Reduced Mean	4·5	2·4	4·2	1·3	3·6	1·5	6·4	7·6	18·5

The "reduced means" are obtained in the same manner as in the case of the previous stars.

The telescopes used were as follows:—

A 68: 1857 June 18, 24, July 31, Aug. 22 to 25, Sept. 7 to 19; 1858 Aug. 6; 1861 May 17 to 1862 Apr. 29; 1862 from July 28; 1863 Apr. 23, May 17, 22, June 20 to 30, Nov. 2; and from 1864 May 13 to 1869 Apr. 30, except 1867 May 5 and 11 and 1868 Apr. 26.

A 40: 1857 June 27, July 25; 1858 July 29; 1859 May 12 to 27 (except 21), June 14 to 22, 30.

T 35: 1857 July 12, and Sept. 22 to Nov. 15.

C 39: 1861 Mar. 19; 1862 July 19, 24; 1863 Feb. 14, May 8, June 12, July 4 to Aug. 17; 1864 Apr. 25; 1867 May 5 and 11; and 1868 Apr. 26.

30 (*a*): 1857 July 1, 11, 16, 19, Aug. 16 to 21, 27 to Sept. 2; 1858 June 12, 26, 27, Aug. 5.

22 (*a*) 21: 1858 July 1 to 17, 19, Aug. 9 to 1859 May 11; 1859 May 21, July 9.

22 (*a*) 28 to 40: 1859 May 28 to June 9, 25, July 5, 19, and throughout 1860.

R (7) 40: 1858 Mar. 16 to May 3, May 21, June 4, 13 to 21, July 18.

R (13) 81: 1858 May 19; 1861 Apr. 13.

E 80: From 1878 June, except 1878 July 31 (6 in. A), and Sept. 13 (E 125).

TABLE XI.

Baxendell's Observations of S Herculis.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857			
June 13	9479	96 − 5	91
14	9480	Visible 95	
15	9481	,, ,,	
17	9483	96 − 6	90
18	9484	89 + 1, 96 − 6	90
24	9490	89 − 1, 96 − 8	88
27	9493	89 − 2, 96 − 11	86
July 1	9497	88 + 0	88
4	9500	88 + 0, 89 − 3	87
11	9507	88 + 0 (difficult).	88
12	9508	88 − 1, 89 − 2	87
16	9512	88 − 1	87
19	9515	88 − 1, 89 − 3	86
20	9516	88 − 1, 83 + 3, 89 − 3	86
25	9521	83 + 2, 88 − 1, 89 − 1	87
31	9527	83 + 1, 88 − 2, 89 − 3	85
Aug. 16	9543	88 − 3, 83 − 2, 81 − 1	82
20	9547	76 + 3, 81 − 2	79
21	9548	76 + 4, 83 − 3	80
22	9549	81 − 1, 83 − 2, 76 + 1	79
23	9550	76 + 2, 83 − 3	79
25	9552	83 − 3	80
27	9554	76 + 3, 83 − 2, 81 − 1	80
28	9555	76 + 3, 83 − 3, 81 − 1	80
Sept. 2	9560	76 + 3, 83 − 3, 81 − 1	80
7	9565	76 + 1, 83 − 3	78
12	9570	76 + 1 (?), 83 − 3, 81 − 2	79
Sept. 16	9574	76 + 1, 83 − 3, 81 − 2	79
19	9577	76 + 1, 83 − 3, 81 − 2	79
22	9580	76 + 1, 83 − 2, 81 − 1	79
25	9583	76 + 2, 83 − 2, 81 − 1	80
28	9586	76 + 2, 81 + 0, 83 − 1	80
Oct. 4	9592	76 + 2, 81 + 0·5, 83 − 0·5	81
6	9594	83 + 1, 88 − 1	86
8	9596	83 + 1, 88 + 0	86
28	9616	Not seen < 83	
Nov. 1	9620	83 + 3, 88 + 0	87
6	9625	83 + 4, 88 + 1	88
15	9634	88 + 2	90
1858			
Jan. 16	9696	Suspected.	
Mar. 6	9745	,,	
16	9755	103 + 6, 106 + 0	108
Apr. 1	9771	Invisible.	
4	9774	103 + 2	105
9	9779	103 + 0	103
12	9782	103 − 1	102
20	9790	103 − 3	100
23	9793	103 − 4	99
May 3	9803	103 − 5	98
19	9819	*88 − 1, 89 + 1	88
21	9821	88 + 0·5, 89 − 0·5	88
June 4	9835	88 + 0, 83 + 3	87
12	9843	83 − 1	82
13	9844	83 + 0, 88 − 2	84
17	9848	83 − 1	82
21	9852	81 + 1, 83 − 2	82

* The designations of the two stars were probably accidentally transposed.

TABLE XI.—*Baxendell's Observations of S Herculis*—continued.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1858			
June 26	9857	81 − 1	80
27	9858	81 − 1	80
July 1	9862	76 + 5, 81 − 2	80
6	9867	76 + 4, 81 − 2	80
10	9871	76 + 4, 81 − 2	80
17	9878	76 + 3, 81 − 2	79
18	9879	76 + 3, 81 − 2	79
19	9880	76 + 4, 81 − 1	80
29	9890	76 + 3, 81 − 1	80
Aug. 5	9897	About 81 + 0	?81
6	9898	81 + 1, 83 − 1	82
9	9901	83 − S=81 + S	82
10	9902	81 + 2, 83 + 0	83
12	9904	81 + 2, 83 + 0	83
21	9913	* About 89 ? + 0	?89
Sept. 5	9928	88 + 2	90
1859	240+		
Feb. 28	0104	? + 0	?
Mar. 31	0135	83 + 0	83
Apr. 3	0138	83 + 0	83
4	0139	83 + 0	83
6	0141	83 + 0	83
9	0144	83 + 0	83
18	0153	83 − 1	82
21	0156	83 − 1	82
26	0161	76 + 1, 83 − 2	79
May 1	0166	76 + 1, 83 − 1	80
2	0167	76 + 1, 83 − 1	80
8	0173	76 + 2, 83 − 1	80
11	0176	76 + 1, 83 − 2	79
12	0177	76 + 1, 83 − 2	79
16	0181	76 + 0, 83 − 2	78
20	0185	76 + 1, 83 − 2	79
21	0186	76 + 1, 83 − 2	79
22	0187	76 + 1, 83 − 3	78
25	0190	76 + 1, 83 − 2	79
May 27	0192	76 + 1, 83 − 2	79
28	0193	76 + 1, 83 − 1	80
29	0194	76 + 2, 83 − 1	80
31	0196	76 + 2, 83 − 2	80
June 7	0203	76 + 3, 83 − 1	80
8	0204	76 + 4, 83 + 0	82
9	0205	76 + 3, 83 − 1	80
14	0210	76 + 4, 83 + 0	82
16	0212	76 + 4, 83 + 0	82
22	0218	76 + 6, 83 + 1	83
25	0221	83 + 3, 88 + 0	87
30	0226	83 + 6, 88 − 2	88
July 5	0231	88 + 5	93
9	0235	88 + 8	96
19	0245	Only glimpsed.	
1860			
Mar. 14	0484	76 + 2, 83 − 3	79
24	0494	76 + 2, 83 − 3	79
Apr. 5	0506	76 + 4, 83 − 3	80
8	0509	76 + 8, 83 + 1	84
11	0512	83 + 2	85
16	0517	83 + 4	87
23	0524	83 + 7, 88 + 4	91
May 13	0544	103 − 3	100
1861			
Mar. 19	0854	Invisible.	
Apr. 13	0879	103 − 6	97
May 17	0913	130	
Oct. 16	1065	81 − 2, †88 − 7	?80 or 79
28	1077	81 − 3, †88 − 7, ?†74 + 10‡	?80 or 78
Nov. 2	1082	†74 + 8, 81 − 3, †88 − 7	?80 or 78
15	1095	†74 + 7, 81 − 3, †88 − 7	?80 or 78

* "Star near w": identification uncertain.
† These identifications are doubtful.
‡ Bax. writes "? if correct."

TABLE XI.—*Baxendell's Observations of S Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1862			
Mar. 3	1203	Invisible.	
Apr. 29	1260	103 + 4	107
July 19	1341	83 + 4, 88 − 2	86
24	1346	83 + 4, 88 − 3	86
28	1350	83 + 3, 88 − 3	86
Aug. 22	1375	*74 + 2, 76 − 3	?74
Sept. 1	1385	*74 + 2, 76 − 2	?75
10	1394	*74 + 2, 76 − 1	?76
15	1399	76 + 2, 81 − 2	78
18	1402	76 + 3, 81 − 2	79
20	1404	76 + 3, 81 − 2	79
Oct. 4	1418	83 + 3, 88 − 3, 89 − 5	85
14	1428	83 + 10, 89 + 4	93
18	1432	88 + 4, 89 + 3	92
1863			
Feb. 14	1551	Invisible < 115	
Apr. 23	1619	89 + 0, 103 − 11	90
May 8	1634	83 + 4, 89 − 3	86
17	1643	83 + 3, 89 − 3	86
22	1648	83 + 3, 89 − 3	86
June 12	1669	76 + 6, 81 + 0	82
20	1677	76 + 0, 81 − 3	77
23	1680	76 + 1, 81 − 5	76
30	1687	76 + 0, 81 − 4	76
July 4	1691	76 + 1, 81 − 3	78
17	1704	76 + 7, 81 − 2, 83 + 1	82
20	1707	83 + 0, 88 − 2	84
Aug. 17	1735	88 + 7, 103 − 8	95
Nov. 2	1812	Glimpsed 128	
1864			
Apr. 25	1987	81 − 2, 76 + 6, 83 − 2	81
May 13	2005	76 + 0	76
26	2018	76 + 3, 83 + 1	82
June 2	2025	83 + 4	87
1864			
July 4	2057	103 + 0	103
Aug. 26	2110	Glimpsed 132	
Sept. 22	2137	,, 129	
1865			
Apr. 10	2337	88 + 0, 89 + 0	88
25	2352	103 − 3	100
May 18	2375	103 + 6	109
24	2381	103 + 7	110
June 9	2397	Not seen < 120	
17	2405	Glimpsed 130	
July 1	2419	Not seen < 132	
14	2432	Just seen est. 125	
21	2439	Limit − 3 ,, 127	
Aug. 25	2474	103 − 1	102
Sept. 7	2487	89 + 7, 103 − 11	94
28	2508	88 + 2, 89 − 1	89
Oct. 7	2517	88 + 3, 89 − 2	89
29	2539	83 + 1	84
Nov. 26	2567	76 + 3, 81 − 4	78
1866			
Apr. 21	2713	Not seen < 120	
May 15	2737	,, < 125	
July 16	2799	89 + 1	90
31	2814	88 + 3, 89 − 2	89
Aug. 27	2841	83 + 2, 88 − 3	85
31	2845	83 + 1, 88 − 2	85
Sept. 17	2862	76 + 2, 83 − 4, 88 − 6	80
Oct. 6	2881	76 + 2, 81 − 2	78
14	2889	76 + 1, 81 − 2	78
Nov. 6	2912	83 + 2, 88 − 2	86
9	2915	83 + 3, 88 − 2	86
30	2936	88 + 9, 103 − 8	96
1867			
May 5	3092	? 103 − 4	?99
11	3098	88 + 7, 103 − 7	96
14	3101	88 + 8, 103 − 7, 89 + 6	96

* These identifications are doubtful.

TABLE XI.—*Baxendell's Observations of S Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1867			
May 21	3108	88 + 5, 89 + 2	92
July 24	3172	76 + 1	77
27	3175	76 + 0	76
Aug. 6	3185	76 − 1	75
Sept. 2	3212	83 + 0	83
16	3226	88 + 3	91
1868			
Apr. 11	3434	76 + 2, 81 − 1, 83 − 2	80
26	3449	76 + 1	77
May 11	3464	74 + 0, 76 − 2	74
20	3473	74 + 0, 76 − 2	74
30	3483	74 + 0, 76 − 2	74
June 4	3488	74 + 1, 76 − 1	75
15	3499	74 + 2, 76 + 0	76
19	3503	74 + 3, 76 + 1	77
July 11	3525	88 − 1, 89 − 3	86
Nov. 15	3652	Not seen < 127.	
1869			
Apr. 30	3818	83 − 1	82
1878			
June 23	7159	89 + 2, 96 − 3	92
28	7164	89 + 0, 96 − 4	90
July 8	7174	88 + 0	88
19	7185	83 − 1	82
28	7194	81 + 2, 83 − 1	82
31	7197	83 + 0	83
Aug. 8	7205	76 + 5, 83 − 2	81
18	7215	76 + 3, 83 − 1	80
21	7218	83 − 1	82
25	7222	83 + 0	83
28	7225	83 + 0	83
Sept. 2	7230	76 + 4, 83 + 0	82
9	7237	83 + 3, 88 − 3	86
11	7239	83 + 3, 88 − 2	86
13	7241	83 + 3, 88 − 2	86
24	7252	89 + 1, 96 − 6, ?83 + 8	90
Oct. 17	7275	96 + 5, 103 − 7	98
26	7284	103 − 2	101
Nov. 1	7290	103 + 1	104

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Aug. 1	7563	?83 + 0	?83
8	7570	?83 + 3, 89 − 1	?87
11	7573	89 + 3, 96 − 3, ?83 + 7	92
14	7576	89 + 2, 96 − 2	92
23	7585	89 + 3, 96 − 3	92
30	7592	96 + 1, 103 − 4	98
Sept. 2	7595	96 + 2, 103 − 4	98
7	7600	103 − 1	102
13	7606	103 + 2	105
25	7618	103 + 8	111
Oct. 5	7628	103 + 9	112
6	7629	103 + 11	114
15	7638	103 + 13, min. vis. − 5 "min. vis. here = 133"	116
Nov. 1	7655	< 103 + 8	
2	7656	103 + 12, min. vis. − 3	115
1880			
Mar. 17	7792	83 − 1, 88 − 3	84
Apr. 1	7807	64 + 1	65
11	7817	64 + 1	65
17	7823	64 + 0	64
May 1	7837	64 + 2	66
9	7845	64 + 2	66
16	7852	83 + 1	84
28	7864	88 − 1	87
July 4	7901	103 + 4	107
20	7917	103 + 11	114
Aug. 9	7937	121 − 2	119
13	7941	121 + 2	123
Sept. 4	7963	Invisible < 103 + 12	
8	7967	,,	
17	7976	,, < 103 + 8	
28	7987	,, < 121 + 3	
Oct. 29	8018	121 − 2	119
Nov. 21	8041	103 + 0	103
1881			
Apr. 1	8172	88 + 1	89
20	8191	96 − 3, 103 − 8	94

TABLE XI.—*Baxendell's Observations of S Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1881			
May 4	8205	103 + 0	103
20	8221	103 + 3	106
June 18	8250	Invisible < 125	
July 14	8276	,, < 130	
Aug. 1	8294	Glimpsed 137	
24	8317	121 − 3	118
Sept. 1	8325	121 − 4	117
14	8338	103 + 7	110
Oct. 2	8356	103 − 3	100
16	8370	96 + 6, 103 − 7	99
20	8374	96 + 0	96
29	8383	96 − 2	94
Nov. 20	8405	83 + 6, 96 − 3	91
1882			
Apr. 8	8544	103 + 0	103
20	8556	103 + 9	112
May 12	8578	121 − 2, min. vis. − 6	119
July 24	8651	103+11, 121 − 6	114
1883			
July 28	9020	76 + 3, 81 − 3, 83 − 3	79
Aug. 25	9048	64 + 1	65
1884			
Mar. 21	9257	121 − 1	120
Apr. 7	9274	About 122	
28	9295	103 − 2	101
May 18	9315	103 − 3	100
22	9319	103 − 4	99
24	9321	103 − 8, 96 − 2	94
June 4	9332	89 + 3, 96 − 4	92
11	9339	96 − 7	89
20	9348	88 − 3, 89 − 4	85
25	9353	83 + 3, 88 − 3	85
July 2	9360	83 − 4, 81 − 3, 76 + 1	78
27	9385	76 + 2, 81 − 3	78
Aug. 1	9390	76 + 2, 81 − 3	78
6	9395	76 + 1, 81 − 3	78
11	9400	76 + 1, 81 − 3, 83 − 3	78
14	9403	76 + 2, 81 − 2	78
19	9408	76 + 4, 81 − 1	80
23	9412	76 + 6, 81 + 1	82
Sept. 5	9425	*83 + 8, 88 − 3	88 or 84
13	9433	89 + 0, 96 − 4	90
20	9440	†89 + 9, 96 − 5	94 or 92
Nov. 19	9500	121 + 0, or 106 + 0	121 or 106
1885			
Apr. 8	9640	103 − 6, 96 − 1	96
17	9649	89 + 0	89
23	9655	†88 + 4, 89 − 1	90 or 88
May 7	9669	83 − 2	81
10	9672	83 − 2	81
20	9682	83 − 2	81
June 1	9694	64 + 5	69
9	9702	64 + 3, 69 − 1	68
July 4	9727	81 + 0, 83 − 2	81
26	9749	83 + 4, 88 + 2	88
Aug. 14	9768	103 − 4,	99
Sept. 5	9790	103 + 3,	106
27	9812	106 − 1	105
1886	241+		
Apr. 9	0006	64 + 2	66
26	0023	64 + 5, 69 + 0	69
May 21	0048	83 + 3, 88 + 0	87
Sept. 17	0167	Glimpsed : say 135	
Oct. 25	0205	103 + 10, 121−11	112
Nov. 2	0213	103 − 2	101
17	0228	89 + 0	89

* ? Error for 76.

† ? Error for 83.

Chandler's formula for this star is

$$2399197 + 308{\cdot}3E + 35 \sin(9E + 86°). \quad \text{(Revision.)}$$

A period of 309 days was assumed, and the observations were arranged and tabulated in the usual manner. The results showed plainly a progressive "change of epoch," not to be accounted for by the period having been taken a little too long. The following means seem to indicate a receding of the place of maximum, amounting to one "portion" (about 26 days) between A and B, and a further receding of the same amount between B and C, and again between C and D. E is in much the same position as D; while F shows an *advance* on D and E of about one "portion." The subscript figures give the number of periods in which the observations occur.

		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A	[I.+(4)]	...	105_1	$94\frac{1}{2}_2$	86_3	82_3	79_4	82_4	$91\frac{1}{2}_4$	$93\frac{1}{2}_2$	...	...	...
B	[VI.+(1)]	...	...	86_1	78_2	79_2	84_2	92_1	...	...	...	...	...
C	[VIII.+(2)]	...	$88\frac{1}{2}_1$	86_1	$79\frac{1}{2}_2$	$79\frac{1}{2}_2$	92_3	106_2	110_1	...	...	102_1	$91\frac{1}{2}_1$
D	[XI.+(4)]	84_3	79_4	78_5	$86\frac{1}{2}_3$	96_1	...	...	...	...	...	97_1	87_3
E	[XXVI.+(4)]	92_2	79_2	$73\frac{1}{2}_2$	84_3	91_3	104_4	$113\frac{1}{2}_3$	117_2	...	118_2	109_3	98_1
F	[XXXII.+(3)]	96_2	87_2	$79\frac{1}{2}_2$	71_3	85_3	92_2	106_1	$105\frac{1}{2}_2$	...	...	116_2	98_2

A mean light curve has been formed by shifting these figures as suggested above, "B" being advanced one "portion," "C" two portions, D and E three portions, and F two portions, and then taking their weighted means. The very much larger and more rapid change in the receding than in the advancing suggests that the assumed period was (as was to be expected) too long. When this shifting was done, one portion (No. 12) was left without any observations, owing to the fact that Baxendell was unable to see the variable when very near its minimum.

The gap being so small, it proved possible to fill in the missing portion with reasonable probability, and so to complete the curve—a very fairly smooth one. This done, the observations were compared with the curve in much the same way as usual. But the amount of variation being so great, it was thought better to use the shifted positions in making the comparisons, in order that the calculated corrections might be small, as it is clear that a correction amounting to more than a full "portion" cannot be safely made in most parts of the curve. It is also obvious that, as the maximum occurs almost exactly in the centre of portion 6, it would be misleading to use that portion in obtaining results, for "$p - f$" has then no real meaning; and also an apparent "correction" might quite probably mean only that the variable was brighter or less bright than the average near that maximum. The portion 6 has therefore always been omitted, and the portion 12 (in which the minimum occurs) had to be omitted in any case, there being no observations available in the shifted positions. The results are given in Table XII. The square brackets show the observations in which the stars "a" and "b'" have been used, wholly or in part.

None of these have been used in obtaining the corrections to epoch. The maximum, as read from the curve, appears to be at about 138 days after the starting point of the period.

TABLE XII.

Comparison of Observations with a Mean Light Curve.

Period.	Initial Date. 239+.	Number and Mean Magnitude of Portion. 1. 113.	2. 102.	3. 93.	4. 86.	5. 80.	6. 77.	7. 83.	8. 92.	9. 102.	10. 110.	11. 117.	12. (119).	
I.	9420	..	...	-4_5	$+1_8$	$+2_4$	$+2_9$	-1_7	-8_2	-10_1	...	...	...	
II.	9729	...	$+3_3$	$+6_4$	$+3_2$	$+3_6$	$+2_7$	-2_6	$-2\cdot5_2$	...	...	...	...	
	240+													
III.	0038	...	...	...	-1_3	$+1_6$	$+2_{10}$	-1_8	-1_5	...	...	...	...	
IV.	0347	...	...	...	...	...	$+2_2$	$+3_5$	$+6_1$	...	...	...	...	
VI.	0939	...	...	...	...	-1_1	$+1_2$	-1_1	...	...	..	...	...	Shift
VII.	1248	$-6\cdot5_1$	...	...	$+1_3$	-4_1	-1_4	$+2_3$	$+3_1$	...	...	...	...	26 days.
VIII.	1531	...	...	...	$+4_2$	$+6_2$	$+1_3$	$-2\cdot5_4$	-1_1	...	...	...	...	Shift
IX.	1840	...	...	...	...	...	$+4_1$	-6_2	-2_1	$+2_1$	...	...	...	52
X.	2149	...	...	...	...	...	...	...	$+1_2$	$+4_1$	$+3_1$	...	...	days.
XI.	2433	...	$+1_1$	-2_2	$+1_1$	$+2_1$	$+1_1$	...	...	...	...	...	...	
XII.	2742	...	...	-4_2	$+1_1$	$+1_2$	$+1_2$	$+1_2$	$+3_1$	...	...	...	...	
XIII.	3051	...	-3_1	-3_1	...	-2_2	-2_1	$+3\cdot5_2$	...	...	...	...	...	
XIV.	3360	...	...	-10_1	-10_1	-6_4	0_2	$+4_1$	..	...	...	..	...	
XV.	3669	...	...	...	...	..	$+5_1$	..	..	.	...	...	...	
XXVI.	7068	...	...	...	$+6_2$	$+4_3$	$+4_4$	$+2_6$	$+1_1$	$+1_3$	...	...	...	Shift
XVII.	7377	...	...	...	...	...	...	...	-4_4	-3_5	$+1_3$	-1_2	...	77
XVIII.	7686	...	...	...	...	$[-6_2]$	$[-12]_3$	$[-3_3]$	...	$+7_2$	$+8\cdot5_2$	...	...	days.
XXIX.	7995	$+10_1$	$+3\cdot5_1$	...	...	...	...	$+2\cdot5_1$	0_1	$+3_2$	...	...	...	
XXX.	8304	$+6_2$	$+6_1$	$+5_3$	$+6_2$	...	...	...	...	...	-3_2	$+1_1$	...	
XXXI.	8638	...	$+12_1$	...	...	...	...	...	...	...	...	..	...	
XXII.	8947	...	...	$[-11]_1$	$[-18]_1$	...	.	...	...	.	...	...	...	
XXIII.	9256	$+3_1$	-1_1	$+3_4$	$+1_3$	-5_1	$+1_6$	$+3_3$	$+2_1$	...	-4_1	...	...	
XXIV.	9565	...	...	$+6_1$	$+1\cdot5_2$	0_2	$[-9]_2$	0_1	$+1_2$	$+2_1$	-6_1	...	...	Shift
XXV.	9874	...	...	...	...	...	$[-10]_2$	$+2_1$	...	...	...	...	...	52
	241+													days.
XXVI.	0183	$+2_1$	-8_2	...	...	...	...	...	...	...	...	...	...	
$o-f$		+17	+20	+16	+13	+9	(−3)	−15	−19	−18	−15	−9	(+4)	

Corrections to epoch have been obtained from each individual period which was sufficiently observed to give some prospect of an approximately correct result. It is, of course, best when both

rise and fall have been observed about equally, but some periods have been included in which this was not the case. The latter results are given in brackets. Table XIII. gives the epochs thus obtained, and compares them with those found by Chandler's formula.

TABLE XIII.

Comparison of Baxendell's Epochs with those of Chandler.

E.	Baxendell. 239+	Chandler.	Bax.–Ch. d.
1	9558+10·9=9568·9	9505·3+34·9=9540·2	+28·7
2	(9867+ 9·9=9876·9)	9813·6+34·0=9847·6	(+29·3)
3	0176+ 1·8=0177·8	0121·9+32·2=0154·1	+23·7
6	1077+ 0·9=1077·9	1046·8+22·5=1069·3	+ 8·6
7	1386– 9·6=1376·4	1355·1+18·0=1373·1	+ 3·3
8	1669+ 9·5=1678·5	1663·4+13·1=1676·5	+ 2·0
9	(1978+ 5·0=1983·0)	1971·7+ 7·9=1979·6	(+ 3·4)
10	(2287– 7·3=2279·7)	2280·0+ 2·4=2282·4	(– 2·7)
11	(2571+ 1·0=2572·0)	2588·3– 3·1=2585·2	(–13·2)
12	2880– 5·1=2874·9	2896·6– 8·5=2888·1	–13·2
13	(3189– 9·2=3179·8)	3204·9–13·7=3191·2	(–11·4)
26	(7206+ 2·0=7208·0)	7212·8–22·5=7190·3	(+17·7)
27	(7515+ 6·2=7521·2)	7521·1–18·0=7503·1	(+18·1)
29	8133+ 4·6=8137·6	8137·7– 7·9=8129·8	+ 7·8
30	(8442+14·4=8456·4)	8446·0– 2·4=8443·6	(+12·8)
33	9394+ 0·9=9394·9	9370·9+13·7=9384·6	+10·3
34	9703+ 4·5=9707·5	9679·2+18·6=9697·8	+ 9·7

The differences between Baxendell and Chandler are considerable, and it is clearly desirable to attempt a revision of his formula. Baxendell's observations are divisible into two groups separated by 13 cycles: a uniform period of 300 days will suit the first, and of 312·5 days the second, as shown in Table IV.

TABLE XIV.

Baxendell's Two Sets Compared with Uniform Periods of 300 Days and 312·5 Days Respectively.

	First Set.			Second Set.			
E.	Obs. Max.	Adopted.	O–C. d.	E.	Obs. Max.	Adopted.	O–C. d.
1	9568·9	9577·0	–8·1	26	(7208·0)	7206·0	(+2·0)
2	(9876·9)	9877·0	(–0·1)	27	(7521·0)	7518·5	(+2·5)
3	0177·8	0177·0	+0·8	29	8137·6	8143·5	–5·9
6	1077·9	1077·0	+0·9	30	(8456·4)	8456·0	(+0·4)
7	1376·4	1377·0	–0·6	33	9394·9	9393·5	+1·4
8	1678·5	1677·0	+1·5	34	9707·5	9706·0	+1·5
9	(1983·0)	1977·0	(+6·0)				
10	(2279·7)	2277·0	(+2·7)				
11	(2572·0)	2577·0	(–5·0)				
12	2874·9	2877·0	–2·1				
13	(3179·8)	3177·0	(+2·8)				

These are the main facts which must ultimately be combined with the results of other observations. From them alone, by a process of trial and error, Miss Blagg deduced the revised formula quoted below. But before giving it, some considerations guiding the choice of such formulæ may be indicated.

There is a considerable difference between the periods for the two sets—approximately 300 days and 312·5 days. If we use for illustration Chandler's term $35 \sin (9^\circ E + 86^\circ) = 35 \sin \theta$ say: then for any value of θ we may measure the correction to period by half the difference of preceding and following values, *i.e.* $35\{\sin(\theta + 9^\circ) - \sin(\theta - 9^\circ)\}/2$ or $35 \sin 9^\circ \cos \theta$. Since $35 \sin 9^\circ = 5{\cdot}5$, the extreme range of periods is 11 days, whereas *at least* 12·5 is indicated above. [The qualification "at least" is added, because in estimating the momentary period over such a long range as from E = 1 to E = 13, there is a tendency to bring it nearer the mean value.] Hence we must be prepared at the outset for an increase either in Chandler's coefficient C = 35 days, or his rate $A = 9^\circ$; or both. The correlation formulæ of *Mon. Not.*, lxviii. p. 544 do not help us in the choice, because both C and A are already larger than their correlation values (C = 27 days and $A = 8^\circ{\cdot}2$).

Next, assuming that the inequality, which we will denote by $C \sin(A^\circ E + B^\circ)$, gives extreme values to the period at E = 7 and E = 30, which are the middle points of the two sets, then we must have $7A + B = 180^\circ$ and $30A + B = 360^\circ$, whence $A = 180^\circ/23 < 8^\circ$; and the mean period should be halfway between the extremes of 300 days and 312·5, or 306·25 days. But the mean period must also fulfil another condition: we may derive it from the difference of mean epochs for the two sets, *i.e.*,

$$\text{period} = (8456{\cdot}0 - 1377{\cdot}0)/23 = 307{\cdot}78 \text{ days.}$$

The difference of 1·6 days would amount to 37 days in 23 periods, and is clearly too large to be neglected in the second method. On the other hand, to increase the value 306·25 by 1·6 days we must increase *each* of the extremes by this amount, if they are to remain extremes. It will be readily seen that this is intolerable in either case. We infer that these values cannot both be extremes, and this will further increase either C or A.

Let us now adopt the following notation. Let the formula be

$$K + PE + C \sin(A^\circ E + B^\circ)$$

P being the period in days, and E the number of periods. Let the value of the angle AE + B be denoted by ϕ_E; so that for Baxendell's observations we are chiefly concerned with ϕ_7 and ϕ_{30}. The comparison of epochs gives us

$$30P + C \sin \phi_{30} - (7P + C \sin \phi_7) = 8456 - 1377$$

$$\text{or} \qquad P = 307{\cdot}78 + C(\sin \phi_7 - \sin \phi_{30})/23 \quad . \qquad (1)$$

We have also two equations for the mean period at E = 7 and E = 30. Consider the first. We have adopted 300 days as sufficiently close to exhibit the nature of the evidence, but we must determine whether we are going to fix attention on the epochs close to E = 7 or on the extremes. A reasonable compromise is to take the mean of the first three residuals and of the last three, giving us between E = 2 and E = 12 a mean period of 300·1 days. The theoretical expression is

$$P + C[\sin(\phi_7 + 5A) - \sin(\phi_7 - 5A)]/10$$

Hence we have

$$10P + 2C \sin 5A \cos \phi_7 = 3001 \quad . \quad . \quad (2)$$

Finally, consideration of the second set gives us

$$6P + 2C \sin 3A \cos \phi_{30} = 1875 \quad . \quad . \quad . \quad (3)$$

These three equations do not suffice in themselves to determine the four quantities, P, A, B, C, but if we assume a series of values for A, we can deduce the other quantities; and it will be a useful guide for other cases to work out the solution with some completeness. Eliminating P, we find two values of C:

$$C = -884/(5 \sin \phi_7 - 5 \sin \phi_{30} + 23 \sin 5A \cos \phi_7) \quad . \quad (4)$$
$$C = +326/(3 \sin \phi_7 - 3 \sin \phi_{30} + 23 \sin 3A \cos \phi_{30}) \quad . \quad (5)$$

Although these are expressed in particular terms instead of general, the significations of the quantities are readily identified for comparison with other cases. Since $\phi_{30} = \phi_7 + 23A$, we ultimately find

$$\tan \phi_7 = \frac{428 \sin 23A - 2033 \sin 3A \cos 23A - 750 \sin 5A}{428(1 - \cos 23A) - 2033 \sin 3A \sin 23A}.$$

For any assumed value of A this gives ϕ_7, and hence all the other quantities. It will suffice to tabulate the following seven solutions.

I.	2399437·0 + 301·5E + 160 sin (4°E + 235°)
II.	9341·5 + 303·6E + 103 sin (5°E + 211)
III.	9294·8 + 305·0E + 74 sin (6°E + 187)
IV.	9267·6 + 305·7E + 59 sin (7°E + 165)
V.	9235·0 + 306·6E + 46 sin (9°E + 122)
VI.	9214·0 + 307·1E + 46 sin (11°E + 80)
VII.	9191·0 + 307·4E + 64 sin (13°E + 38)

Miss Blagg's "trial and error" solution was

$$2399222·0 + 307·3E + 45 \sin(9°E + 110),$$

which is intermediate between V. and VI. These seven solutions are compared with the observed epochs of Table XIV. in Table XV.

TABLE XV.

Residuals of the Different Solutions for S Herculis.

E.	I. 4°.	II. 5°.	III. 6°.	IV. 7°.	V. 9°.	VI. 11°.	VII. 13°.
1	−16·5	−15·7	−14·2	−12·6	−7·4	+ 1·8	+20·8
2	− 4·5	− 4·2	− 3·8	− 3·1	−0·9	+ 3·7	+13·3
3	− 0·5	− 0·4	− 0·7	− 0·9	−0·7	+ 0·1	+ 2·3
6	+ 5·0	+ 4·9	+ 3·6	+ 2·6	+0·1	− 4·4	−15·0
7	+ 3·7	+ 3·8	+ 2·5	+ 1·6	−0·8	− 5·3	−16·1
8	+ 5·3	+ 5·6	+ 4·3	+ 3·8	+1·6	− 1·9	−11·1
9	+ 8·5	+ 9·0	+ 8·0	+ 7·6	+6·6	+ 4·3	− 1·7
10	+ 3·1	+ 4·0	+ 3·1	+ 3·1	+3·1	+ 2·7	+ 1·4
11	− 7·4	− 6·3	− 7·1	− 6·5	−5·4	− 3·6	+ 0·7
12	− 8·3	− 6·8	− 7·2	− 6·4	−4·1	+ 0·1	+10·6
13	− 7·7	− 6·1	− 6·3	− 5·1	−1·6	+ 4·9	+21·7
26	+ 5·3	+ 4·5	+ 4·8	+ 4·6	+4·6	+ 5·2	+ 6·9
27	+ 6·2	+ 5·2	+ 5·3	+ 4·8	+3·8	+ 1·9	− 0·8
29	− 1·9	− 3·1	− 3·5	− 4·4	−6·8	−10·9	−21·4
30	+ 4·3	+ 3·1	+ 2·6	+ 1·6	−1·0	− 5·8	−15·9
33	+ 4·9	+ 4·2	+ 3·4	+ 3·5	+3·3	+ 0·9	− 1·5
34	+ 5·0	+ 4·7	+ 4·6	+ 4·9	+5·5	+ 6·2	+ 9·5

Now Table XV. shows that if we selected a solution from Baxendell's observations alone, we should probably take something between A = 9° and A = 11°, as Miss Blagg did. But the choice must ultimately be guided by the agreement of other observations, and we have already the Rousdon observations available (*Mem. R.A.S.*, vol. lv. p. lxxxv, Table xliii.). When these pages were printed off, Chandler's Revision had not been received, and only his 3rd Catalogue was available, which gave a mean period 308^{d}·1, with the note "large irregularities." From the Rousdon observations mean periods were deduced as follows:—

$$\text{at E} = 41 \quad P = 310^{d}\cdot3 \qquad E = 46 \quad P = 307^{d}\cdot6,$$

the Rousdon maximum for E = 44 being quoted as 2412779.

Chandler's Revision was received just before going to press with vol. lv.; and in an Appendix (p. cxiv) it is shown that the changes from the 3rd Catalogue are all in accordance with the Rousdon suggested corrections, "with the single exception of S Herculis": the point being that the Rousdon mean period (309·0 days) agreed better with the 308·1 days of the 3rd Catalogue than with the revision formula

$$308\cdot3E + 35 \sin (9^{\circ}E + 86^{\circ}),$$

which, when E = 44, gives a period $308{\cdot}3 + 5{\cdot}5 \cos 122^\circ = 305{\cdot}5$. Indeed, since the Rousdon observations indicate a period greater than the mean, the argument $44A + B$ must lie between 270° and 450°, and cannot have such a value as 122° just found.

Now for the seven solutions tabulated above we have the following values for $44A + B$:—

I.	II.	III.	IV.	V.	VI.	VII.
51°	71°	91°	113°	158°	204°	250°

Of these IV., V., VI. all give a period sensibly less than the mean at the Rousdon epoch, and must be excluded, unless the Rousdon observations are seriously wrong. No. VII. is approaching 270°, and suggests that we ought to increase A still further—say to A = 14°; but it is seen from Table XV. that many of the residuals are already large for A = 13°, and would become intolerable for any further advance in the same direction.

There remain the solutions I., II., III. We may exhibit their accordance with the Rousdon observations most concisely by calculating the maxima for E = 39, 44, and 49, from which the above deductions (for Rousdon) were made.

TABLE XVI.

Maxima from Solutions I., II., III., and Deduced Periods.

	I.		II.		III.	
E.	Maxima.	Periods.	Maxima.	Periods.	Maxima.	Periods.
39	2411267·9	311·9	1242·5	311·0	1254·6	306·8
44	2827·3	306·9	2797·3	304·6	2788·8	302·9
49	4361·9		4320·4		4303·2	
Mean	2818·7	309·4	2786·7	307·8	2782·2	304·8

All three solutions show a decrease in period similar to the Rousdon 310·3 and 307·0 above quoted; but none of them are quite large enough, even the first. It would seem from these periods as though we should decrease A even below 4°.

The Rousdon maximum for E = 44 above quoted is 2412779, which, on the other hand, seems as though A should exceed 6°. But this maximum was fixed from the adopted curve without any attempt at correction. If we solve each line of the mean residuals on p. lxxxvi (*loc. cit.*) for corrections to epoch, we obtain

for "adopted 1069" or E = 39, correction = − 6·2
" " 2609" or E = 44, " = + 7·7
" " 4149" or E = 49, " = + 4·8,

so that a small positive mean correction is indicated. The adoption of a definitive formula may now be left until other observations are being discussed; but it seems pretty clear that the value of A is only about half that assigned by Chandler, or

the "correlation" formula: possibly it may be *exactly* half the more usual value.

The light curve, when analysed, gives the following expression in magnitudes, the arbitrary starting point being the centre of portion 1, or 13 days after the initial point:

$$1{\cdot}997\cos(\theta - 318^\circ) + 0{\cdot}044\cos 2(\theta - 116^\circ{\cdot}2) + 0{\cdot}134\cos 3(\theta - 94^\circ{\cdot}8).$$

For comparison with the Rousdon result we must divide by the range (4·2 mags.) and multiply by 6; this expression becomes:

$$2{\cdot}85\cos(\theta - 318^\circ) + 0{\cdot}07\cos 2(\theta - 116^\circ{\cdot}2) + 0{\cdot}19\cos 3(\theta - 94^\circ{\cdot}8).$$

This expression has its greatest possible negative value when θ is about $145\frac{1}{2}^\circ$. This is equal to about 125 days after the zero of the expression, or 138 after the starting point of the period; exactly in agreement with the position read off from the curve. It is not possible to form similar expressions for groups of periods, as none of these form complete curves.

Referred to maximum as starting point, the expression becomes

$$2{\cdot}85\cos(\theta - 173^\circ) + 0{\cdot}07\cos 2(\theta - 151^\circ) + 0{\cdot}19\cos 3(\theta - 72^\circ).$$

The Rousdon curve is (*Mem. R.A.S.*, vol. iv. p. xcix)

$$2{\cdot}80\cos(\theta - 170^\circ) + 0{\cdot}33\cos 2(\theta - 126^\circ) + 0{\cdot}21\cos 3(\theta - 35^\circ).$$

Note on the Comparison of Two Plates of the same Region in the Vatican Zones of the Astrographic Catalogue. By Ethel F. Bellamy.

(*Communicated by Professor Turner.*)

1. It has been mentioned that the plates for the Astrographic zones $+55^\circ$ to $+64^\circ$ which have been taken by Fr. Lais of the Vatican Observatory, and measured at Rome by ladies of a religious order, are being reduced at Oxford (R.A.S. Council Report, *M.N.*, lxxiii. p. 246).

2. In the course of this work it was noticed that the plate labelled $23^h\ 36^m$ was a repetition in error for $23^h\ 52^m$, so that this latter region had been taken and measured twice. As there was an interval of 14·10 years between the dates, it seemed desirable to compare the two sets of measures, in order (1) to test the general accordance of the Rome measures; (2) to detect any sensible P.Ms. in this region.

Plate $779 + 61^\circ$ $23^h\ 52^m$ was exposed 1899·737.
„ $780 + 61^\circ$ $23^h\ 52^m$ „ „ 1913·832.

3. The plates were both reduced in the usual manner, by a comparison of the measures with those stars in Helsingfors A.G.C.,

14390–14671, brought to epoch 1900·0, and the plate constants deduced were as follows:—

	A.	B.	C.	D.	E.	F.
Plate 779 (= P_1)	+ ·00343	+ ·00005	− ·0393	− ·00011	+ ·00356	− ·1543
,, 780 (= P_2)	+ ·00349	+ ·00219	− ·0613	− ·00227	+ ·00356	− ·0951

4. From these constants, linear formulæ were obtained for converting the earlier set of measures to the later set, viz.:—

$$x_2 - x_1 = + \cdot00006x_1 + \cdot00215y_1 - \cdot0220,$$
$$y_2 - y_1 = - \cdot00217x_1 + \cdot00000y_1 + \cdot0594.$$

The distribution of the resulting differences is shown in Table I.

TABLE I.

Distribution of Differences $(x_2 - x_1)$ *and* $(y_2 - y_1)$.

x/y.	−6.	−5.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.	+6.	Totals.
+6	...	...	...	...	..	1	...	...	...	...	...	...	1	2
+5	...	1	...	...	1	2	...	2	...	1	3	...	...	10
+4	...	...	1	...	...	5	7	6	3	3	2	2	1	30
+3	...	...	...	3	12	16	20	21	9	10	3	1	1	96
+2	1	1	4	6	16	28	24	27	19	11	4	2	1	144
+1	...	1	2	6	28	28	47	41	27	15	5	3	...	203
0	3	1	...	9	13	47	41	40	33	19	9	4	3	222
−1	...	1	2	11	19	31	34	23	18	11	3	5	2	160
−2	...	...	2	9	13	11	13	10	8	5	2	2	...	75
−3	...	...	...	1	4	10	6	9	7	3	3	...	...	43
−4	..	...	1	...	2	2	2	5	2	5	2	1	...	22
−5	...	1	...	1	2	2	1	...	3	1	2	...	...	13
−6	...	...	...	...	1	...	...	...	...	...	...	...	...	1
Totals	4	6	12	46	111	183	195	184	129	84	38	20	9	1021

Besides these there are:—

x.	y.	x.	y.	x.	y.
−8	−3	+1	−8	+7	+1
−3	+7	+2	+8	+7	+1
−1	−8	+2	+7	+8	+4
−1	−8	+3	+7	+8	0
+1	+7	+5	−8	+9	−4

It may be here remarked that the quantities given in this paper are expressed in units of the thousandth of a réseau interval, ·001 = 0″·3.

5. The "mean of the errors" (η) in x was found to be $\pm 1\cdot71$ and in y $\pm 1\cdot60$. The "mean of the squares" (σ) was found to be $\pm 2\cdot14$ and $\pm 2\cdot07$.

These give for the probable error (restoring for a moment the réseau interval as unit)

	In x.	In y.
$r=0\cdot845\eta$	$= \pm\cdot00144$	$\pm\cdot00135$
$r=0\cdot674\sigma$	$= \pm\cdot00144$	$\pm\cdot00140$

These being differences, the probable error of a single measure may be taken as $\pm\cdot0010$ or $\pm 0''\cdot3$. This is rather larger than usual (probably for the reason mentioned just below), and as, moreover, it seemed probable that some at least of the larger differences were due to accidental errors of measurement, application was made to Fr. Hagen for the loan of these plates, to which he kindly and promptly acceded.

6. Upon receipt of them in February, all cases with a difference between the two plates amounting to 5 or larger, 72 in number, were remeasured. It was soon seen that owing to the thickness of the réseau lines on plate 779 there would be a greater probable error of measurement, and that most of the differences resulting from the first comparison of these two plates would be due to this cause. Indeed, after scrutiny of the stars with these differences (5 or over), there were only nine cases of sensible difference remaining for examination on the second plate.

7. After the later plate 780 had been examined, the following stars (Table II.) were left in as the residue. The number of stars and amount of probable P.Ms. are small, and the comparison of these two plates thus affords chiefly negative evidence of the existence of any large P.Ms.

Table II.

Provisional No. in Vatican $+61^\circ$ 23^h 52^m.	Mag. (photo.).	A.G.C. or [B.D.].	Δx in 14·1 Years.	Δy in 14·1 Years.	Centennial P.M. in Arc.	Remarks.
90	10·1	[$+59^\circ$ 2779]	−9	−1	19″·3	Elong. and on the réseau line.
1103	10·9	...	−1	+7	15·1	
22	10·9	...	−6	+3	14·3	Corner of the plate.
36	12·2	...	+5	+4	13·6	
226	12·2	...	−6	+1	13·0	Corner of the plate.
115	12·2	...	+4	+3	10·7	Very faint.
1091	12·7	...	+3	+3	9·0	
643	10·9	...	−1	+4	8·8	
942	12·2	...	0	+3	6·4	
1229	8·4	14447	+3	−4	10·7	

The number in Column I. is provisional, being taken from the unprinted MS. of the Vatican measures; Column II. has the photographic magnitude as deduced from the measured diameter d with formula $14\cdot7 - 1\cdot02\sqrt{d}$ found from a comparison of the

brighter stars to the 9·5 mag. in Helsingfors A.G.C. corrected to the Harvard scale.

Column III. shows that none of the Helsingfors stars show a P.M. of the magnitude regarded as sensible on the evidence of the original measures.

The one star included, Hels. 14447 (epoch 1877·0), is below the adopted limit, the Δx and Δy being $+3$ and -4; but it has been included on the additional evidence afforded by the collected residuals as entered in the ledger, viz.:—

	x.	y.	Plate Centre. h m °
1899	+ 7	− 9	23 52 +61
1903	+ 6	− 8	23 52 +63
1904	+11	− 7	23 42 +62
1913	+10	−13	23 52 +61

Column VI. is the centennial P.M. in arc, deduced from Δx and Δy in Columns IV. and V., as given by the revised comparison of the two plates, in units of ·001 or 0″·3.

8. In Boss's *General Catalogue of 6188 Stars* he gives P.Ms. for the following Helsingfors stars: 14425, 14603, 14641, 14657, and 14406; but they would all give differences in x and y less than ·001 of a réseau interval in 14 years, and the comparison of the above two plates does not afford any confirmation of much value.

9. It may be mentioned that the correlation of the differences $x_2 - x_1$ and $y_2 - y_1$ was calculated from Table I., the result being $r = ·040 \pm ·021$, too small to be significant.

10. Notwithstanding the large probable error (which must be principally due to the thick réseau lines on the earlier plate), there is evidence of great care in making the measures, of which one instance may be given. A small double star with components each of about 10·5 magnitude and with co-ordinates (revised) on 2nd plate

$$x = 12·523 \qquad y = 11·284$$
$$x = 12·522 \qquad y = 11·266$$

(*i.e.* separation 0″·3 in x and 5″·4 in y) is clearly separated on the second plate, as one might expect. But on the first plate the images are so diffused as to form only a slightly elongated patch. Nevertheless, the double character was recognised and measured. The reason of this diffusion on the earlier plate seems to be that the central parts are out of focus, for which there is confirmatory evidence, as follows:—

(*a*) The plate constants give a small difference in x (though not in y), corresponding to a position of the plate nearer the O.G. at the earlier date.

(*b*) The central images generally are larger on the earlier plate.

(*c*) A number of stars measured on the first plate, but missed on the second (and some of them impossible to find even on scrutiny), are all near the borders, where they would be fainter on the second plate.

University Observatory, Oxford:
1915 *March* 8.

Occultations of the Pleiades, 1914 Dec. 1 and 28. By H. C. Plummer, M.A., Royal Astronomer of Ireland.

Occultations of the Pleiades by the Moon were observed with the South equatorial (aperture $11\frac{3}{4}$ inches) on 1914 Dec. 1 and 28. On the first date the times were recorded on the chronograph; on the second the times were taken with a chronometer, compared with the standard sidereal clock before and after the observations.

The co-ordinates of Dunsink Observatory given in the *Nautical Almanac* refer to the position of the meridian circle. The equatorial is situated about 289 feet east and about 99 feet south of this point. The sidereal times, which define the R.A. of the zenith, are therefore corrected by the amount $+0^s.32$. In the reductions the geocentric latitude and $\log \rho$ have been taken from Bauschinger's *Tafeln* and are therefore based on Bessel's figure; the subtraction of $1''.0$ gives $\phi' = 53^\circ\ 12'\ 10''.2$.

The places of the stars A.G.C. (Berlin B) 1138 and 1160 have been taken from that catalogue without proper motion. The position of A.G.C. (Berlin B) 1187 was taken from Astronomical Papers of the American Ephemeris, vol. viii. part iii., and reduced to mean place accordingly. The places of the other stars have been taken directly from the *Nautical Almanac*. The data relative to the Moon, contained in the last four columns of Tables I. and II., are derived by interpolation from the *Nautical Almanac* (and therefore include Newcomb's corrections).

The equations of condition are based on the following notation:—

Star's apparent R.A.	$= A + 1''.\ \Delta A$
,, ,, decl.	$= D + 1''.\ \Delta D$
Moon's ,, R.A.	$= \alpha + 1''.\ \Delta\alpha$
,, ,, decl.	$= \delta + 1''.\ \Delta\delta$
,, parallax	$= p + 1''.\ \Delta p$
,, semi-diameter	$= s + 1''.\ \Delta s$
Geocentric latitude	$= \phi' + 1''.\ \Delta\phi'$
W. longitude	$= l + 1^s.\ \Delta l$

The quantities A, D, α, δ, p, s are given in Tables I. and II.; the adopted value of ϕ' is given above. The adopted longitude l has also been explained. If T is the actual time of an observation, the true time should have been $T + 1^s.\ \Delta T$.

A rough preliminary solution gives, for the corrections to the Moon's tabular place—

1914 Dec. 1:	$\Delta\alpha = +0^s.71$,	$\Delta\delta = +5''.6$
Dec. 28:	$+0\ .76$	$+1\ .5$

But all other terms are here neglected, and combination with other observations is desirable.

Dunsink Observatory:
1915 *March* 11.

TABLE I. 1914 Dec. 1.

No.	Name of Star.	Phase.	Sidereal Time.	Star's R.A.	Star's Decl.	Moon's R.A.	Moon's Decl.	Moon's Parallax.	Moon's Semi-diameter.
			h m s	h m s	° ′ ″	h m s	° ′ ″	′ ″	′ ″
1	17 Tauri	Dis.	22 21 1·39	3 39 51·00	+23 51 0·6	3 36 25·65	+24 29 19·4	54 9·06	14 46·82
2	16 Tauri	,,	22 24 34·49	3 39 46·36	+24 1 34·2	3 36 33·20	+24 29 44·6	54 9·08	14 46·83
3	A.G.C. (Berlin B) 1138	,,	22 47 56·68	3 40 35·68	+24 4 27·2	3 37 22·66	+24 32 29·6	54 9·25	14 46·87
4	19 Tauri	,,	22 54 10·98	3 40 10·18	+24 12 17·4	3 37 35·86	+24 33 13·4	54 9·29	14 46·88
5	20 Tauri	,,	22 54 58·98	3 40 47·46	+24 6 22·6	3 37 37·56	+24 33 19·0	54 9·30	14 46·88
6	22 Tauri	,,	23 18 2·87	3 41 0·39	+24 16 0·6	3 38 26·42	+24 36 0·5	54 9·47	14 46·93
7	21 Tauri	,,	23 22 2·46	3 40 51·95	+24 17 35·9	3 38 34·88	+24 36 28·4	54 9·50	14 46·94
8	A.G.C. (Berlin B) 1160	,,	23 36 14·46	3 41 56·80	+24 15 39·8	3 39 4·99	+24 38 7·3	54 9·60	14 46·97
9	20 Tauri	Reapp.	23 59 8·04	3 40 47·46	+24 6 22·6	3 39 53·56	+24 40 45·9	54 9·77	14 47·01
10	A.G.C. (Berlin B) 1187	Dis.	0 20 30·03	3 43 27·74	+24 5 18·5	3 40 38·93	+24 43 13·0	54 9·93	14 47·06

TABLE II. 1914 Dec. 28.

11	16 Tauri	Dis.	9 34 13·02	3 39 46·44	+24 1 34·9	3 41 28·90	+24 52 31·9	54 16·56	14 48·87
12	20 Tauri	,,	9 55 50·38	3 40 47·55	+24 6 23·3	3 42 14·97	+24 54 56·7	54 16·75	14 48·92
13	16 Tauri	Reapp.	10 5 32·30	3 39 46·44	+24 1 34·9	3 42 35·65	+24 56 1·4	54 16·84	14 48·94
14	A.G.C. (Berlin B) 1160	Dis.	10 22 25·95	3 41 56·90	+24 15 40·5	3 43 11·69	+24 57 53·6	54 17·00	14 48·99
15	19 Tauri	Reapp.	10 31 47·87	3 40 10·26	+24 12 18·2	3 43 31·67	+24 58 55·4	54 17·08	14 49·01
16	20 Tauri	,,	10 40 38·20	3 40 47·55	+24 6 23·3	3 43 50·54	+24 59 53·8	54 17·16	14 49·03

Observers: (1–8) H. C. P.; (9–16) C. Martin.

Equations of Condition.

1914 Dec. 1.

(1)	− 8·58	= − ·907Δα	+ ·076Δδ	+ ·905ΔA	− ·081ΔD	− ·442ΔT	− ·472Δ*l*	+ ·012Δφ′	− ·642Δ*p*	− 1 . Δ*s*
(2)	− 10·54	= − ·739	− ·584	+ ·740	+ ·575	− ·471	− ·461	+ ·016	− ·083	− 1
(3)	− 9·72	= − ·784	− ·508	+ ·783	+ ·499	− ·468	− ·476	+ ·015	− ·163	− 1
(4)	− 7·38	= − ·264	− ·957	+ ·267	+ ·948	− ·292	− ·253	+ ·013	+ ·455	− 1
(5)	− 10·79	= − ·757	− ·555	+ ·756	+ ·546	− ·460	− ·467	+ ·016	− ·115	− 1
(6)	− 9·98	= − ·330	− ·932	+ ·332	+ ·922	− ·312	− ·284	+ ·014	+ ·384	− 1
(7)	− 9·04	= − ·111	− ·992	+ ·114	+ ·983	− ·219	− ·175	+ ·012	+ ·550	− 1
(8)	− 10·18	= − ·642	− ·708	+ ·641	+ ·698	− ·411	− ·423	+ ·016	+ ·058	− 1
(9)	+ 10·67	= + ·896	+ ·166	− ·902	− ·163	+ ·583	+ ·496	− ·012	+ ·388	− 1
(10)	− 8·89	= − ·809	+ ·454	+ ·802	− ·454	− ·278	− ·379	+ ·003	− ·675	− 1

1914 Dec. 28.

(11)	− 3·56	= − ·558Δα	+ ·788Δδ	+ ·561ΔA	− ·782ΔD	− ·253ΔT	− ·210Δ*l*	− ·015Δφ′	− ·200Δ*p*	− 1 . Δ*s*
(12)	− 8·12	= − ·760	+ ·546	+ ·763	− ·541	− ·384	− ·345	− ·015	+ ·096	− 1
(13)	+ 2·93	= + ·380	+ ·908	− ·377	− ·906	+ ·258	+ ·304	− ·002	− ·935	− 1
(14)	− 10·59	= − ·906	+ ·026	+ ·908	− ·023	− ·508	− ·482	− ·013	+ ·570	− 1
(15)	+ 10·78	= + ·870	+ ·283	− ·870	− ·288	+ ·510	+ ·496	+ ·010	− ·784	− 1
(16)	+ 9·78	= + ·629	+ ·720	− ·628	− ·722	+ ·401	+ ·416	+ ·003	− ·972	− 1

Double Stars measured at the Cape (First Series). By J. Voûte.

(Communicated by H.M. Astronomer at the Cape.)

The following stars have been measured with the 7-inch refractor; Merz objective, mounting by Troughton & Simms, and Repsold micrometer.

With a few exceptions all the position-angles are the means of measures with and without reversing prism, as proposed by "Bosler & Salet." *

For bright stars the objective has been covered with a screen.

More particulars about the measures will be given later.

Innes's Reference Catalogue.	Star's Name.	Epoch 1900+.	Position Angle.	Distance.	Nights.
			°	"	
0.33	β 395	14·963	109·2	0·70	3
1.1	β Phœnicis	13·930	9·2	1·60	4
...	,,	14·956	8·2	1·52	4
1.13	Innes 27	14·979	270·4	0·68	3
1.14	κ Toucani = *h* 3423	14·979	348·0	5·54	3
1.37	*p* Eridani = Dunlop 5	13·962	214·4	8·57	4
...	H III 80	13·962	293·8	12·18	2
4.2	Russell 38	13·963	245·1	2·17	4
...	Σ 517	14·942	76·7	8·20	1
4.13	Gale	13·937	353·9	0·84	4
4.18	Σ 536	13·905	166·9	1·86	2
4.44	*h* 3683	13·962	77·4	1·14	4
5.3	γ Cæli = Jacob 2	13·924	311·2	3·24	2
5.5	Hargrave 2	14·010	169·1	1·14	4
5.14	β Orionis = Σ 668	13·905	203·2	9·52	2
5.84	Jacob 3	14·040	175·8	2·88	4
6.3	*h* 3834	14·093	224·1	4·16	4
6.4	Dunlop 23	14·057	56·2	1·72	4
6.18	Dunlop 26	14·036	117·43	21·06	4
...	*h* 3861	14·113	71·9	2·08	2
6.54	A.G.C. 1 = Sirius	14·015	79·6	10·24	4
7.2	Dunlop 39	14·087	79·7	2·36	2
...	Dunlop 41	14·113	44·8	7·15	2
7.83	Howe 8	14·097	283·8	3·06	2
8.89	H Velorum = Russell 87	14·181	340·5	2·90	4

* My method C in Leiden double-star measures, J. Voûte, "Doppelsternmessungen," *Annalen der Leidener Sternw.*, Bd. ix.

Innes's Reference Catalogue.	Star's Name.	Epoch 1900+.	Position Angle.	Distance.	Nights.
			°	″	
8.96	*h* 4165	14·196	101·4	1·27	4
9.40	ψ Argus = Copeland 1	14·268	123·7	1·39	4
9.47	*h* 4220	14·272	208·8	2·18	4
10.55	β 411	14·331	273·2	0·56	3
10.62	t_2 Carinæ = Dunlop 94	14·345	20·1	14·48	4
,,	Lac. 4401 = Gilliss	14·378	74·2	20·78	2
...	η Argus	14·424	75·3	0·80	4
10.74	μ Argus = Russell 155	14·363	67·0	2·32	4
10.83	*h* 4383	14·420	284·2	1·94	2
11.19	*h* 4432	14·422	298·5	2·43	4
11.22	Brisbane	14·326	69·2	1·99	4
11.31	Innes 78	14·448	91·8	1·12	2
11.44	Cape 11	14·419	218·9	2·68	4
11.60	Washb. 114	14·391	193·4	2·41	4
12.19	Rümker 14	14·463	243·5	3·20	4
12.61	γ Centauri = *h* 4539	14·472	348·1	1·33	4
12.68	β Muscæ = Russell 207	14·460	351·8	1·25	4
14.3	Sellors 19	14·532	254·7	0·95	4
14.59	α Centauri	13·967	217·32	17·65	6
...	,,	14·503	218·10	17·53	6
14.63	α Circini = Dunlop 166	14·516	235·5	15·66	4
14.105	So and *h* 190	14·540	296·42	17·94	4
14.116	π Lupi = *h* 4728	14·535	84·0	1·43	4
15.72	Howe 37	14·616	346·2	4·08	2
15.92	*h* 4813	14·572	100·7	4·02	4
15.94	A Scorpii = β 36	14·600	274·8	2·66	4
15.116	ξ Scorpii = Σ 1998	14·590	346·8	0·97	4
16.12	c_1 Scorpii = *h* 4839	14·606	75·5	4·19	4
16.13	Jacob 9	14·583	49·2	2·29	4
16.14	ν Scorpii = β 120	14·583	2·5	1·08	4
17.21	A Ophiuchi = So and *h* 243	14·605	185·0	4·32	4
17.31	Lac. 7194 = Brisbane	14·594	149·5	2·68	4
17.35	Melbourne 4 = β 416	14·600	249·4	2·10	6
17.52	*h* 4949	14·628	260·2	2·54	4
17.106	Cape 18	14·652	91·6	2·64	2
17.123	τ Ophiuchi = Σ 2262	14·627	260·2	2·35	4
17.129	*h* 5014	14·624	52·8	1·50	4
18.113	γ Coronæ Australis	14·662	100·5	2·42	4

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

The following measures were made with the 8-inch Coleman telescope belonging to the Society. The stars were selected as being of special interest on account of their binary character or relative motion.

The magnitudes of the Σ pairs have been taken from vol. lvi. of the *Memoirs* of the Society. The places are those of the same volume corrected for precession.

In the last column, A = H. F. Acocks; P = T. E. R. Phillips.

Star.	1910 R.A.	1910 Dec.	Mags.	Position Angle.	Distance.	No. of Nights.	Date.	Observer.
	h m	° ′		°	″			
Σ 60 (η Cassiopeiæ)	0 43·6	+57 20	4·0, 7·6	250·0	6·64	3	1914·654	P
73 (36 Androm.)	0 50·8	23 8	6·2, 6·8	41·0	0·80	3	·703	P
82	0 55·9	9 1	8·3, 9·3	305·7	1·73	3	·697	A
113	1 15·2	− 0 59	6·2, 7·2	356·5	1·53	3	·714	P
122	1 22·3	+ 3 4	7·9, 9·0	329·3	5·83	3	·736	P
138	1 31·3	7 11	7·3, 7·3	42·1	1·54	2	·693	A
158	1 41·5	32 43	8·3, 8·8	258·1	2·07	3	·738	P
178	1 47·3	10 22	7·8, 7·8	198·9	2·98	2	·694	A
180 (γ Arietis)	1 48·5	18 51	4·2, 4·4	359·3	8·25	2	·995	P
185	1 53·8	75 3	7·0, 8·5	21·2	1·23	3	·949	P
186	1 51·2	1 24	7·2, 7·2	39·5	1·03	3	1915·001	P
202 (α Piscium)	1 57·4	2 20	2·2, 3·9	315·0	2·55	3	1914·828	P
205 (γ Androm.) BC	1 58·4	41 54	2·5, 5·5	115·4	0·55	3	·833	P
262 (ι Cassiop.) AB	2 21·6	67 0	4·2, 7·1	250·6	2·12	3	·951	P
BC	...	...	4·2, 8·1	111·8	7·40	3	·951	P
305	2 42·4	19 0	7·3, 8·2	314·9	3·29	3	·824	P
333 (ε Arietis)	2 54·1	20 59	5·7, 6·0	203·9	1·42	3	·824	P
425	3 34·3	33 52	7·3, 7·3	89·2	2·42	3	·852	P
535	4 18·3	11 10	6·7, 8·2	326·4	1·27	3	·871	P
577	4 36·1	37 10	7·7, 7·7	59·8	1·37	3	·887	P
OΣ 95	5 0·2	19 41	6·5, 7	324·6	0·78	3	·898	P
98	5 3·0	8 23	6 7	150·0	0·86	3	·904	P
Σ 644	5 4·2	37 12	6·7, 7·0	220·4	1·59	3	·885	P
η Orionis	5 19·9	− 2 29	4 5	80·8	1·20	3	·948	P
Σ 716	5 23·5	+25 5	5·8, 6·6	201·4	4·66	3	·897	P
749	5 31·5	26 52	7·0, 7·1	347·3	0·90	3	·898	P
881 (4 Lyncis)	6 14·1	59 25	6·4, 7·9	113·0	0·81	3	·926	P
918	6 26·8	52 32	6·9, 7·8	330·4	4·74	3	·926	P
945	6 34·0	41 4	7·1, 8·0	268·6	0·73	3	·931	P
948 (12 Lyncis) AB	6 38·1	59 31	5·2, 6·1	112·2	1·63	3	·936	P
AC	...	...	5·2, 7·4	306·3	8·48	2	·943	P

Star.	1910 R.A. h m	1910 Dec. ° ′	Mags.	Position Angle. °	Distance. ″	No. of Nights.	Date.	Observer.
982 (38 Geminorum)	6 49·6	13 18	5·4, 7·7	157·0	6·50	3	1914·931	P
1110 (Castor)	7 28·9	32 6	2·7, 3·7	219·3	5·04	6	·973	P
1126	7 35·3	5 27	7·2, 7·5	148·9	1·19	3	·971	P
1130	7 36·8	9 55	8·4, 8·9	177·5	1·02	3	·971	P
1196 (ζ Cancri) AB	8 7·1	17 55	5·0, 5·5	308·6	0·97	3	1915·000	P
AC	...	...	5·0, 5·5	112·7	4·76	3	·000	P
OΣ 215	10 11·5	18 12	6·5, 7	24·2	1·08	3	1914·192	P
Σ 1356 (ω Leonis)	9 23·7	9 26	6·2, 6·7	122·9	1·07	3	·160	P
1424 (γ Leonis)	10 15·0	20 18	2·0, 3·5	117·4	3·74	3	·175	P
1457	10 34·0	6 12	7·6, 8·5	321·0	1·24	3	·207	P
1536 (ι Leonis)	11 19·2	11 2	3·9, 7·1	45·9	2·05	3	·160	P
1670 (γ Virginis)	12 37·0	− 0 57	3·0, 3·0	325·4	5·97	3	·160	P
1687 (35 Comæ Ber.)	12 48·9	+21 44	5·2, 8·0	89·3	1·11	3	·290	P
1781	13 41·6	5 34	7·8, 8·2	283·0	0·95	3	·313	P
1819	14 10·8	3 33	7·9, 8·0	344·8	1·23	2	·378	A
1865 (ζ Böotis)	14 36·9	14 6	3·5, 3·9	141·4	0·75	5	·264	P
1888 (ξ Böotis)	14 47·2	19 29	4·7, 6·6	110·2	2·22	4	·295	P
1909	15 0·9	48 0	5·2, 6·1	244·6	3·90	3	·314	P
1937 (η Coronæ)	15 19·5	30 37	5·5, 6·0	51·4	0·99	3	·259	P
2055 (λ Ophiuchi)	16 26·3	2 11	4·0, 6·1	70·5	1·08	3	·381	P
2130 (μ Draconis)	17 3·5	54 36	5·0, 5·1	131·9	2·27	3	·365	P
2272 (70 Ophiuchi)	18 1·0	2 32	4·5, 6·0	142·8	4·25	4	·479	P
2323	18 22·5	58 41	4·7, 7·7	355·5	3·45	3	379	P
2384	18 38·5	67 3	8·0, 8·5	308·3	0·82	3	·453	P
2583 (π Aquilæ)	19 44·4	11 35	6·1, 6·7	114·2	1·36	3	·865	P
2672	20 17·7	23 29	8·7, 8·8	298·3	0·77	3	·676	P
OΣ 413 (λ Cygni)	20 43·9	36 10	5, 6	52·7	0·73	3	·443	P
Σ 2895	22 16·5	24 39	8·5, 10·0	35·4	8·16	3	·645	A
				37·5	8·52	3	·741	P
2909 (ζ Aquarii)	22 29·1	− 0 30	4·0, 4·1	307·1	3·02	2	·608	A
2944 AB	22 43·2	4·42	7·0, 7·5	260·9	3·11	3	·668	A
3041 BC	23 43·2	+16 32	7·3, 8·1	179·7	3·40	3	·699	P
3050	23 55·0	33 14	6·0, 6·0	222·4	2·15	3	·669	P
3060	0 1·4	17 34	8·7, 8·7	122·3	3·71	3	·689	P
3061	0 1·1	17 20	8·0, 8·0	147·8	7·65	4	·689	P

Notes.

Σ 205 BC (γ^2 Androm.). Pair neatly divided.

Σ 1110 (Castor). The distance seems rather small, but the six sets of measures agree very closely. The pair is perhaps now closing.

Σ 1130. The angle has increased about 15°, and the distance decreased 1″ since the earlier measures were made. The change, however, seems to have been much accelerated in recent years, and the pair is probably a binary system.

Σ 1888 (ξ Böotis). The Comes is getting away from the apparent orbit in vol. lvi. of the *Memoirs*, and measures are now important.

On a General Solution of Hill's Equation. By E. Lindsay Ince, M.A., B.Sc., Research Student in the Mathematical Laboratory, University of Edinburgh.

(*Communicated by E. T. Whittaker.*)

§ 1. *Introduction.*—The differential equation of G. W. Hill,

$$\frac{d^2y}{dz^2} + (\Theta_0 + 2\Theta_1 \cos 2z + 2\Theta_2 \cos 4z + \ . \ . \ .)y = 0 \ * \quad . \quad (1)$$

may be regarded as an extended case of Mathieu's equation,

$$\frac{d^2y}{dz^2} + (a + 16q \cos 2z)y = 0. \dagger \quad . \quad . \quad . \quad (2)$$

In it y may be taken to represent the extent of departure from the periodic orbit caused by the action of various perturbative forces, and z as the time-variable. The quantities Θ_0, Θ_1, Θ_2, etc., are merely known numerical constants.

From the general theory of linear differential equations with periodic coefficients, it is known ‡ that the general solution of both (1) and (2) is of the form

$$y = Ae^{\mu z}\phi(z) + Be^{-\mu z}\psi(z),$$

where A and B are arbitrary constants, μ is a constant depending solely on the Θ_0, Θ_1, Θ_2, etc., in the first, and on a and q in the second case, and ϕ and ψ are purely periodic functions of z, with period equal to that of the coefficients in the differential equation.

In general, μ is not zero, but it may so happen that the numerical constants (Θ_0, Θ_1, Θ_2, etc., or a and q) of the differential equation are such that μ does vanish, in which case ϕ and ψ become identical and y is expressed as a purely periodic function of z. Thus in this case, not the general solution, but a single solution is known, to which a second and quasi-periodic solution is related as in the "logarithmic case" of Bessel functions.

In the case of Mathieu's equation these purely periodic solutions are known ;§ in particular, if

$$a = 1 + 8q - 8q^2 - 8q^3 + \ . \ . \ .$$

the equation (2) has a solution

$$y = \sin z + q \sin 3z + q^2(\sin 3z + \tfrac{1}{3} \sin 5z) + q^3(\tfrac{1}{3} \sin 3z + \tfrac{4}{9} \sin 5z + \tfrac{1}{18} \sin 7z) + \ . \ . \ . \quad . \quad . \quad (3)$$

* Hill, Cambridge, U.S.A., 1877 ; *Acta Mathematica*, vol. viii. pp. 1–36 (1886) ; and *Collected Works*, i. pp. 243–270.

† *v.* Bruns, *Ast. Nach.*, No. 2533, S. 193–204 (1883), and No. 2553, S. 129–132 (1884).

‡ *v.* Floquet, *Annales de l'École Normale Supérieure*, (2), T. 12, pp. 47–88 (1883).

§ *v.* Whittaker, *Proceedings of the International Congress of Mathematicians*, vol. i., Cambridge, 1912.

and if $$a = 1 - 8q - 8q^2 + 8q^3 + \ldots$$

the equation (2) has a solution

$$y = \cos z + q\cos 3z + q^2(-\cos 3z + \tfrac{1}{3}\cos 5z) + q^3(\tfrac{1}{3}\cos 3z - \tfrac{4}{9}\cos 5z + \tfrac{1}{18}\cos 7z) + \ldots \quad (4)$$

The very close resemblance in form which exists between these two solutions suggested to Whittaker that they might be mere degenerate cases of a general solution of Mathieu's equation having the form

$$y = e^{\mu z}u,$$

where

$$u = \sin(z-\sigma) + a_3\cos(3z-\sigma) + b_3\sin(3z-\sigma) + a_5\cos(5z-\sigma) + b_5\sin(5z-\sigma) + \ldots$$

where σ was a new parameter depending solely on the constants a and q of the differential equation (2). The solution (3) above would then correspond to the case $\sigma = 0$, and the solution (4) to $\sigma = \frac{\pi}{2}$.* In this expression the coefficient of $\sin(z-\sigma)$ is taken to be unity, and no term whatsoever in $\cos(z-\sigma)$ appears. By means of this stipulation, σ is completely determined—it amounts, in fact, to a definition of σ. It would, of course, be equally possible to express u as a series beginning with $\cos(z-\sigma)$ in which no term in $\sin(z-\sigma)$ appears, in which case σ would be differently defined.

Acting on this suggestion, Whittaker found that Mathieu's equation was actually satisfied by an expression of the form $y = e^{\mu z}u(z)$, where $u(z)$ is a purely periodic function of z of the above form and μ is given by the power-series in q,

$$\mu = 4q\sin 2\sigma - 12q^3\sin 2\sigma - 12q^4\sin 4\sigma + \ldots \quad (5)$$

the parameter σ being connected with the constants a and q of the differential equation by the relation

$$a = 1 + 8q\cos 2\sigma + (-16 + 8\cos 4\sigma)q^2 - 8q^3\cos 2\sigma + \ldots \quad (6)$$

and the coefficients a_3, b_3, a_5, b_5, etc., of the Fourier series for u are themselves power-series in q with coefficients involving the parameter σ, viz.,

$$b_3 = q + q^2\cos 2\sigma + (-\tfrac{14}{3} + 5\cos 4\sigma)q^3 + \ldots$$
$$a_3 = 3q^2\sin 2\sigma + 3q^3\sin 4\sigma + \ldots$$
$$b_5 = \tfrac{1}{3}q^2 + \tfrac{4}{9}q^3\cos 2\sigma + \ldots$$
$$a_5 = \tfrac{14}{9}q^3\sin 2\sigma + \ldots$$

and so on.

In the astronomical problem of the mean motion of the Moon's

* Whittaker, *Proc. Edin. Math. Soc.*, xxxii. (1913–14), p. 76.

perigee which led Hill to the discussion of the differential equation known by his name, the principal aim is to evaluate the quantity μ on which depends the secular departure from the truly periodic orbit. This is a matter of considerable difficulty, because no simple relation can be found expressing μ directly in terms of the constants Θ_0, Θ_1, Θ_2 . . . Hill himself solved the problem by the use of an infinite determinant, a method previously unknown in analysis, and obtained μ numerically to fifteen places of decimals (sixteen significant figures). It is the object of the present paper to apply Whittaker's method of solving Mathieu's equation to Hill's equation, and thus to obtain literal expressions for μ in terms of σ, and for the parameter Θ_0 in terms of σ and the constants Θ_1, Θ_2, etc., of the equation, by means of which expressions μ might be evaluated. The process itself is really equivalent to introducing a new parameter σ in place of the old parameter Θ_0, and then obtaining a simple relation between μ and this new parameter and the constants of the equation, from which we may ultimately deduce for μ its numerical value.

§ 2. *The Method of Solution.*—As in the case of Mathieu's equation, we assume that the general solution of Hill's equation is

$$y = e^{\mu z} u,$$

where

$$\mu = p_1(\sigma)\Theta_1 + p_2(\sigma)\Theta_2 + \ . \ . \ . \ + q_1(\sigma)\Theta_1^2 + q_2(\sigma)\Theta_2^2 + \ . \ . \ .$$
$$+ q_{12}(\sigma)\Theta_1\Theta_2 + q_{13}(\sigma)\Theta_1\Theta_3 + q_{23}(\sigma)\Theta_2\Theta_3 + \ . \ . \ . \ + r_1(\sigma)\Theta_1^3 + \ . \ . \ .$$

and

$$u = \sin(z-\sigma) + A_1(z,\sigma)\Theta_1 + A_2(z,\sigma)\Theta_2 + \ . \ . \ . \ + B_1(z,\sigma)\Theta_1^2$$
$$+ B_2(z,\sigma)\Theta_2^2 + \ . \ . \ . \ + B_{12}(z,\sigma)\Theta_1\Theta_2 + \ . \ . \ . \ + C_1(z,\sigma)\Theta_1^3 + \ . \ . \ .$$

σ being determined by the relation

$$\Theta_0 = 1 + \lambda_1(\sigma)\Theta_1 + \lambda_2(\sigma)\Theta_2 + \ . \ . \ . \ + \mu_1(\sigma)\Theta_1^2 + \mu_2(\sigma)\Theta_2^2 + \ . \ . \ .$$
$$+ \mu_{12}(\sigma)\Theta_1\Theta_2 + \ . \ . \ . \ + \nu_1(\sigma)\Theta_1^3 + \ . \ . \ .$$

Substituting these expressions in the differential equation, we get

$$-\sin(z-\sigma) + A_1''\Theta_1 + A_2''\Theta_2 + \ . \ . \ . \ + B_1''\Theta_1^2 + B_2''\Theta_2^2 + \ . \ . \ .$$
$$+ B_{12}''\Theta_1\Theta_2 + B_{13}''\Theta_1\Theta_3 + \ . \ . \ . \ + C_1''\Theta_1^3 + \ . \ . \ .$$
$$+ C_{12}''\Theta_1^2\Theta_2 + \ . \ . \ . \ + C_{21}''\Theta_1\Theta_2^2 + \ . \ . \ .$$
$$+ 2(p_1\Theta_1 + p_2\Theta_2 + \ . \ . \ . \ + q_1\Theta_1^2 + q_2\Theta_2^2 + \ . \ . \ . \ + q_{12}\Theta_1\Theta_2 + \ . \ . \ .$$
$$+ r_1\Theta_1^3 + \ . \ . \ .)\{\cos(z-\sigma) + A_1'\Theta_1 + A_2'\Theta_2 + \ . \ . \ . \ + B_1'\Theta_1^2$$
$$+ B_2'\Theta_2^2 + \ . \ . \ . \ + B_{12}'\Theta_1\Theta_2 + \ . \ . \ .\}$$
$$+ \{(p_1\Theta_1 + p_2\Theta_2 + \ . \ . \ . \ + q_1\Theta_1^2 + \ . \ . \ .)^2 + 1 + \lambda_1\Theta_1 + \lambda_2\Theta_2 + \ . \ . \ .$$
$$+ \mu_1\Theta_1^2 + \mu_2\Theta_2^2 + \ . \ . \ . \ + \mu_{12}\Theta_1\Theta_2 + \ . \ . \ . \ + \nu_1\Theta_1^3 + \ . \ . \ .$$
$$+ 2\Theta_1\cos 2z + 2\Theta_2\cos 4z + \ . \ . \ .\}\{\sin(z-\sigma) + A_1\Theta_1 + A_2\Theta_2 +$$
$$. \ . \ . \ + B_1\Theta_1^2 + B_2\Theta_2^2 + \ . \ . \ . \ + B_{12}\Theta_1\Theta_2 + \ . \ . \ .\} = 0.$$

The terms independent of $\Theta_1, \Theta_2 \ldots$ in this equation are identically zero. If we equate to zero the terms involving nothing but the first power of Θ_1, we get

$$A_1'' + A_1 + 2p_1 \cos(z-\sigma) + 2 \sin(z-\sigma) \cos 2z + \lambda_1 \sin(z-\sigma) = 0$$

or

$$A_1'' + A_1 + 2p_1 \cos(z-\sigma) + \lambda_1 \sin(z-\sigma) + \sin(3z-\sigma) - \sin(z-\sigma) \cos 2\sigma - \cos(z-\sigma) \sin 2\sigma = 0.$$

It has already been observed that the expression for u is to contain no term in $\cos(z-\sigma)$. We may conveniently take the coefficient of $\sin(z-\sigma)$ in it to be unity, which simply amounts to fixing the arbitrary constant by which the solution is multiplied. Such being the case, $A_1, A_2 \ldots B_1, B_2 \ldots B_{12} \ldots$ and so on, contain no terms in either $\cos(z-\sigma)$ or $\sin(z-\sigma)$.

We thus have, in the case of Θ_1,

$$p_1 = \tfrac{1}{2} \sin 2\sigma.$$
$$\lambda_1 = \cos 2\sigma.$$
$$A_1 = \tfrac{1}{8} \sin(3z-\sigma).$$

Proceeding in this way we may obtain as many coefficients of the terms in the expression for u as we may desire. We shall not obtain an expression for the general term of the series, but from the computing point of view this is a matter of no consequence.

§ 3. *The Solution in detail.*—The terms in the expansions, so far as they have been worked out, are entered below, under the power or product of the Θ's to which they correspond. The small Roman letters, p, q, $r \ldots$ refer to the coefficients in the expansion of μ; Greek letters, $\lambda, \mu, \nu \ldots$ refer to the expansion of Θ_0; and capital Roman letters, A, B, C $\ldots$ to the expansion of u.

Terms in Θ_1—

$$p_1 = \tfrac{1}{2} \sin 2\sigma.$$
$$\lambda_1 = \cos 2\sigma.$$
$$A_1 = \tfrac{1}{8} \sin(3z-\sigma).$$

Terms in Θ_r, *where* $r = 2, 3, 4 \ldots$—

$$p_r = 0.$$
$$\lambda_r = 0.$$
$$A_r = \frac{1}{4r(r+1)} \sin\{(2r+1)z - \sigma\} - \frac{1}{4r(r-1)} \sin\{(2r-1)z + \sigma\}$$

Terms in Θ_1^2—

$$q_1 = 0.$$
$$\mu_1 = -\frac{1}{4} + \frac{1}{8} \cos 4\sigma.$$
$$B_1 = \frac{1}{192} \sin(5z-\sigma) + \frac{3}{64} \sin 2\sigma \cos(3z-\sigma) + \frac{1}{64} \cos 2\sigma \sin(3z-\sigma).$$

Terms in Θ_r^2, *where* $r = 2, 3, 4 \ldots$—

$$q_r = 0.$$

$$\mu_r = -\frac{1}{2(r^2-1)}.$$

$$B_r = \frac{1}{32r^2(r+1)(2r+1)} \sin\{(4r+1)z-\sigma\} - \frac{1}{32r^2(r-1)(2r-1)} \sin\{(4r-1)z+\sigma\}.$$

Terms in $\Theta_1\Theta_2$—

$$q_{12} = \frac{1}{8} \sin 2\sigma.$$

$$\mu_{12} = \frac{1}{4} \cos 2\sigma.$$

$$B_{12} = \frac{1}{288} \sin(7z-\sigma) + \frac{1}{288} \sin 2\sigma \cos(5z-\sigma) - \frac{1}{288} \cos 2\sigma \sin(5z-\sigma) - \frac{1}{32} \sin 4\sigma \cos(3z-\sigma) + \left(\frac{1}{48} - \frac{1}{32} \cos 4\sigma\right) \sin(3z-\sigma).$$

Terms in $\Theta_1\Theta_r$, *where* $r = 3, 4, 5 \ldots$—

$$q_{1r} = 0.$$

$$\mu_{1r} = 0.$$

$$B_{1r} = \frac{r^2+r+2}{32r(r+1)^2(r+2)} \sin\{(2r+3)z-\sigma\} + \frac{r^2-r-1}{8r^2(r+1)(r^2-1)} \sin 2\sigma \cos\{(2r+1)z-\sigma\} - \frac{1}{8r^2(r+1)(r^2-1)} \cos 2\sigma \sin\{(2r+1)z-\sigma\} - \frac{1}{16r(r-1)^2} \sin 4\sigma \cos\{(2r-1)z-\sigma\} + \frac{1}{16r(r-1)} \left\{ \frac{r+2}{r(r+1)} - \frac{\cos 4\sigma}{r-1} \right\} \sin\{(2r-1)z-\sigma\} - \frac{r^2-r+2}{32r(r-1)^2(r-2)} \sin\{(2r-3)z+\sigma\}.$$

Terms in $\Theta_r\Theta_{r+1}$, *where* $r = 2, 3, 4 \ldots$—

$$q_{r,\,r+1} = \frac{1}{4r(r+1)} \sin 2\sigma.$$

$$\mu_{r,\,r+1} = \frac{1}{2r(r+1)} \cos 2\sigma.$$

$$B_{r,\,r+1} = \frac{1}{16r(r+1)(r+2)(2r+1)} \sin\{(4r+3)z-\sigma\}$$
$$- \frac{1}{16r(r^2-1)(2r+1)} \sin\{(4r+1)z+\sigma\}$$
$$+ \frac{r^2+r+1}{16r(r^2-1)(r+2)} \sin(3z-\sigma).$$

Terms in $\Theta_r\Theta_s$, where $r=2, 3, 4 \ldots$ and $s>r+1$—

$$q_{rs} = 0.$$
$$\mu_{rs} = 0.$$
$$B_{rs} = \frac{1}{16(s+r)(s+r+1)} \left\{ \frac{1}{r(r+1)} + \frac{1}{s(s+1)} \right\} \sin\{(2r+2s+1)z-\sigma\}$$
$$- \frac{1}{16(s+r)(s+r-1)} \left\{ \frac{1}{r(r-1)} + \frac{1}{s(s-1)} \right\} \sin\{(2r+2s-1)z+\sigma\}$$
$$+ \frac{1}{16(s-r)(s-r+1)} \left\{ \frac{1}{r(r-1)} + \frac{1}{s(s+1)} \right\} \sin\{(2s-2r+1)z-\sigma\}$$
$$- \frac{1}{16(s-r)(s-r-1)} \left\{ \frac{1}{r(r+1)} + \frac{1}{s(s-1)} \right\} \sin\{(2s-2r-1)z+\sigma\}.$$

Terms in Θ_1^3—

$$r_1 = -\frac{3}{128} \sin 2\sigma.$$

$$\nu_1 = -\frac{1}{64} \cos 2\sigma.$$

$$C_1 = \frac{1}{9216} \sin(7z-\sigma) + \frac{7}{2304} \sin 2\sigma \cos(5z-\sigma)$$
$$+ \frac{1}{1152} \cos 2\sigma \sin(5z-\sigma) + \frac{3}{512} \sin 4\sigma \cos(3z-\sigma)$$
$$+ \left(\frac{5}{512} \cos 4\sigma - \frac{7}{768}\right) \sin(3z-\sigma).$$

Terms in Θ_r^3, where $r=2, 3, 4 \ldots$—

$$r_r = 0.$$
$$\nu_r = 0.$$
$$C_r = \frac{1}{384r^3(r+1)(2r+1)(3r+1)} \sin\{(6r+1)z-\sigma\}$$
$$- \frac{1}{384r^3(r-1)(2r-1)(3r-1)} \sin\{(6r-1)z+\sigma\}$$
$$- \frac{7r^2+4r+1}{128r^3(r+1)^2(r^2-1)(2r+1)} \sin\{(2r+1)z-\sigma\}$$
$$+ \frac{7r^2-4r+1}{128r^3(r-1)^2(r^2-1)(2r-1)} \sin\{(2r-1)z+\sigma\}.$$

Terms in $\Theta_1^2\Theta_2$—

$$r_{12} = \frac{1}{128}\sin 4\sigma.$$

$$\nu_{12} = -\frac{11}{192} + \frac{5}{64}\cos 4\sigma.$$

$$C_{12} = \frac{1}{9216}\sin(9z-\sigma) + \frac{43\sin 2\sigma}{27648}\cos(7z-\sigma) + \frac{\cos 2\sigma}{3072}\sin(7z-\sigma)$$
$$-\frac{11}{6912}\sin 4\sigma\cos(5z-\sigma) + \left(\frac{5}{13824} - \frac{7}{6912}\cos 4\sigma\right)\sin(5z-\sigma)$$
$$+\left(\frac{119}{4608}\sin 2\sigma - \frac{1}{128}\sin 6\sigma\right)\cos(3z-\sigma)$$
$$+\left(\frac{119}{9216}\cos 2\sigma - \frac{1}{128}\cos 6\sigma\right)\sin(3z-\sigma).$$

Terms in $\Theta_1^2\Theta_3$—

$$r_{13} = \frac{5}{384}\sin 2\sigma.$$

$$\nu_{13} = \frac{5}{192}\cos 2\sigma.$$

Terms in $\Theta_1^2\Theta_r$, $r>3$—

$$r_{1r} = 0.$$
$$\nu_{1r} = 0.$$

Terms in $\Theta_r^2\Theta_{2r-1}$—

$$r_{r,2r-1} = \frac{3r-2}{32r^2(r-1)^2(2r-1)}\sin 2\sigma.$$

$$\nu_{r,2r-1} = \frac{3r-2}{16r^2(r-1)^2(2r-1)}\cos 2\sigma.$$

Terms in $\Theta_r^2\Theta_{2r}$—

$$r_{r,2r} = 0.$$

$$\nu_{r,2r} = -\frac{3}{4(r^2-1)(4r^2-1)}.$$

Terms in $\Theta_r^2\Theta_{2r+1}$—

$$r_{r,2r+1} = \frac{3r+2}{32r^2(r+1)^2(2r+1)}\sin 2\sigma.$$

$$\nu_{r,2r+1} = \frac{3r+2}{16r^2(r+1)^2(2r+1)}\cos 2\sigma.$$

Terms in $\Theta_r^2\Theta_s$, except for $\begin{cases} s = 2r - 1 \\ s = 2r \\ s = 2r + 1 \\ s = 1 \end{cases}$

$$r_{rs} = 0.$$
$$\nu_{rs} = 0.$$

Terms in $\Theta_1\Theta_2^2$—

$$r_{21} = \frac{7}{288}\sin 2\sigma.$$

$$\nu_{21} = -\frac{1}{144}\cos 2\sigma.$$

$$\begin{aligned} C_{21} = {} & \frac{23}{691200}\sin(11z - \sigma) + \frac{\sin 2\sigma}{14400}\cos(9z - \sigma) \\ & - \frac{\cos 2\sigma}{14400}\sin(9z - \sigma) - \frac{1}{1152}\sin 4\sigma\cos(7z - \sigma) \\ & + \left(\frac{7}{11520} - \frac{1}{1152}\cos 4\sigma\right)\sin(7z - \sigma) \\ & + \frac{19}{9216}\sin 2\sigma\cos(5z - \sigma) + \frac{1}{3072}\cos 2\sigma\sin(5z - \sigma) \\ & - \frac{1}{128}\sin 4\sigma\cos(3z - \sigma) + \left(\frac{1}{576} - \frac{1}{128}\cos 4\sigma\right)\sin(3z - \sigma). \end{aligned}$$

Terms in $\Theta_1\Theta_r^2$—

$$r_{r1} = \frac{2r^2 - 1}{8r^2(r-1)^2(r+1)^2}\sin 2\sigma.$$

$$\nu_{r1} = -\frac{1}{4r^2(r-1)^2(r+1)^2}\cos 2\sigma.$$

Terms in $\Theta_1\Theta_2\Theta_3$—

$$r_{123} = \frac{1}{1152}\sin 4\sigma.$$

$$\nu_{123} = \frac{13}{576}\cos 4\sigma - \frac{13}{192}.$$

Terms in $\Theta_1\Theta_2\Theta_4$—

$$r_{124} = \frac{1}{384}\sin 2\sigma.$$

$$\nu_{124} = \frac{1}{192}\cos 2\sigma.$$

Terms in Θ_1^4—

$$s_1 = -\frac{3}{1024}\sin 4\sigma.$$

$$\varpi_1 = \frac{1}{48} - \frac{11}{512}\cos 4\sigma.$$

$$D_1 = \frac{1}{737280}\sin(9z-\sigma) + \frac{35}{442368}\sin 2\sigma\cos(7z-\sigma) + \frac{\cos 2\sigma}{49152}\sin(7z-\sigma)$$
$$+ \frac{11\sin 4\sigma}{27648}\cos(5z-\sigma) + \left(\frac{41\cos 4\sigma}{55296} - \frac{155}{221184}\right)\sin(5z-\sigma)$$
$$+ \left(\frac{9\sin 6\sigma}{4096} - \frac{137\sin 2\sigma}{18432}\right)\cos(3z-\sigma)$$
$$+ \left(\frac{7\cos 6\sigma}{4096} - \frac{37\cos 2\sigma}{18432}\right)\sin(3z-\sigma).$$

Terms in Θ_r^4—

$$s_r = 0.$$
$$\varpi_r = \frac{7r^2+5}{32(r^2-1)^3(4r^2-1)}.$$

Terms in $\Theta_1^3\Theta_2$—

$$s_{12} = -\frac{211}{9216}\sin 2\sigma + \frac{5}{1024}\sin 6\sigma.$$
$$\varpi_{12} = -\frac{175}{9216}\cos 2\sigma + \frac{7}{512}\cos 6\sigma.$$

Terms in Θ_1^5—

$$t_1 = \frac{137}{36864}\sin 2\sigma - \frac{9}{8192}\sin 6\sigma.$$
$$\rho_1 = \frac{1}{288}\cos 2\sigma - \frac{13}{4096}\cos 6\sigma.$$

Terms in Θ_1^6—

$$v_1 = \frac{337}{442368}\sin 4\sigma - \frac{15}{65536}\sin 8\sigma.$$
$$\sigma_1 = -\frac{893}{221184} + \frac{9181}{1769472}\cos 4\sigma - \frac{35}{32768}\cos 8\sigma.$$

§ 4. *Method of Verification.*—A simple check upon the correctness of the above results, and a means of obtaining further terms in the expansions for Θ_0 and μ, may easily be derived from the original differential equation.

As we have seen, Hill's equation has a solution $y = e^{\mu z}\phi(z)$, where $\phi(z)$ is periodic in z and of period 2π.

Now consider the more simple case of Hill's equation,

$$\frac{d^2y}{dz^2} + (\Theta_0 + 2\Theta_1\cos 2z + 2\Theta_2\cos 4z)y = 0,$$

and try to satisfy it by an expression of the form

$$y = e^{\mu z}\left\{\begin{array}{r}\sin(z-\sigma) + b_3\sin(3z-\sigma) + b_5\sin(5z-\sigma) + \ldots \\ + a_3\cos(3z-\sigma) + a_5\cos(5z-\sigma) + \ldots\end{array}\right\}$$

the a's and b's being independent of z.

We thus find that the following relations must hold:—

$$\mu = \tfrac{1}{2}\Theta_1 \cos 2\sigma - \tfrac{1}{2}a_3\Theta_1 - \tfrac{1}{2}a_5\Theta_2 - \tfrac{1}{2}a_3\Theta_2 \cos 2\sigma + \tfrac{1}{2}b_3\Theta_2 \sin 2\sigma,$$

and

$$\Theta_0 = 1 + \Theta_1 \cos 2\sigma - \mu^2 - b_3\Theta_1 + a_3\Theta_2 \sin 2\sigma - b_5\Theta_2 + b_3\Theta_2 \cos 2\sigma$$

which represent μ and Θ_0 so far as they involve only Θ_1 and Θ_2. Knowing a_3, b_3, etc., in the general expansion, we may use the above relations as a verification of our results, or even in some cases to determine unknown coefficients in either or both of the expansions of μ and Θ_0.

§ 5. *Series for* Θ_0 *and* μ.—As the purpose of this paper is to indicate a possible method for computing μ, rather than to carry out such a computation in its fullest details, an accuracy of ten places of decimals only will be aimed at. Since in Hill's equation Θ_i is a quantity of the $2i$th order, the following expansions for Θ_0 and μ will be sufficient for our purpose, viz.,

$$\begin{aligned}\Theta_0 = 1 &+ \Theta_1 \cos 2\sigma + \frac{1}{4}\Theta_1\Theta_2 \cos 2\sigma + \frac{1}{12}\Theta_2\Theta_3 \cos 2\sigma + \left(-\frac{1}{4} + \frac{1}{8}\cos 4\sigma\right)\Theta_1^2 \\ &- \frac{1}{6}\Theta_2^2 - \frac{1}{64}\Theta_1^3 \cos 2\sigma + \left(-\frac{11}{192} + \frac{5}{64}\cos 4\sigma\right)\Theta_1^2\Theta_2 \\ &+ \frac{5}{192}\Theta_1^2\Theta_3 \cos 2\sigma - \frac{1}{144}\Theta_1\Theta_2^2 \cos 2\sigma + \left(\frac{13}{576}\cos 4\sigma - \frac{13}{192}\right)\Theta_1\Theta_2\Theta_3 \\ &+ \left(\frac{1}{48} - \frac{11}{512}\cos 4\sigma\right)\Theta_1^4 + \left(-\frac{175}{9612}\cos 2\sigma + \frac{7}{512}\cos 6\sigma\right)\Theta_1^3\Theta_2 \\ &+ \left(\frac{1}{288}\cos 2\sigma - \frac{13}{4096}\cos 6\sigma\right)\Theta_1^5 \\ &+ \left(-\frac{893}{221184} + \frac{9181}{1769472}\cos 4\sigma - \frac{35}{32768}\cos 8\sigma\right)\Theta_1^6.\end{aligned}$$

$$\begin{aligned}\mu = \frac{1}{2}\Theta_1 &\sin 2\sigma + \frac{1}{8}\Theta_1\Theta_2 \sin 2\sigma + \frac{1}{24}\Theta_2\Theta_3 \sin 2\sigma - \frac{3}{128}\Theta_1^3 \sin 2\sigma \\ &+ \frac{1}{128}\Theta_1^2\Theta_2 \sin 4\sigma + \frac{5}{384}\Theta_1^2\Theta_3 \sin 2\sigma + \frac{7}{288}\Theta_1\Theta_2^2 \sin 2\sigma \\ &+ \frac{1}{1152}\Theta_1\Theta_2\Theta_3 \sin 4\sigma - \frac{3}{1024}\Theta_1^4 \sin 4\sigma \\ &+ \left(-\frac{211}{9216}\sin 2\sigma + \frac{5}{1024}\sin 6\sigma\right)\Theta_1^3\Theta_2 \\ &+ \left(\frac{137}{36864}\sin 2\sigma - \frac{9}{8192}\sin 6\sigma\right)\Theta_1^5 \\ &+ \left(\frac{337}{442368}\sin 4\sigma - \frac{15}{65536}\sin 8\sigma\right)\Theta_1^6.\end{aligned}$$

These, of course, reduce, when $\Theta_1 = 8g$ and $\Theta_2 = \Theta_3 = \Theta_4 = \ldots = 0$, to the expressions (6) and (5) previously given for the a and μ of Mathieu's equation respectively.

It is noteworthy that of terms of the first degree, that in Θ_1 alone appears; product terms of the second degree appear only in the form $\Theta_r\Theta_{r+1}$, and no terms whatsoever in Θ_r^2 exist in the expansion for μ. In addition, it may be remarked that there are no terms in either $\Theta_2\Theta_4$ or $\Theta_1^2\Theta_4$, which is important in view of the fact that the order of such a term is that of Θ_1^6, which is retained in the above expansions.

§ 6. *Numerical Computation of* μ.—The actual numerical values of the quantities Θ_0, Θ_1, Θ_2, etc., which enter into Hill's equation are as follows (to eleven places of decimals):—

$$\Theta_0 = 1{\cdot}15884\ 39396\ 0$$
$$\Theta_1 = -{\cdot}05704\ 40187\ 5$$
$$\Theta_2 = \ {\cdot}00038\ 32380\ 0$$
$$\Theta_3 = -{\cdot}00000\ 91732\ 9.$$

So far as the present work is concerned, Θ_4 and higher Θ's are not required, as they do not give rise to terms whose order is less than the twelfth.

Using then the above values, we find that the expression for Θ_0 becomes

$$1{\cdot}15884\ 39396\ 0 = 1 - {\cdot}00081\ 33804\ 9 - {\cdot}05704\ 65855\ 7\cos 2\sigma + {\cdot}00040\ 66226\ 27\cos 4\sigma + {\cdot}00000\ 00009\ 45\cos 6\sigma - {\cdot}00000\ 00000\ 368\cos 8\sigma.$$

whence

$$\cos 2\sigma = -2{\cdot}79871\ 82492\ 1 + {\cdot}00712\ 79047\ 3\cos 4\sigma + {\cdot}00000\ 00165\ 65\cos 6\sigma - {\cdot}00000\ 00064\ 51\cos 8\sigma.$$

As there are no tables available giving circular functions of a complex argument to a sufficient degree of accuracy, it is found best to treat the above equation as an algebraic equation in $\cos 2\sigma$. If, therefore, we make the substitution $x = \cos 2\sigma$, the equation becomes

$$1{\cdot}00000\ 00496\ 9x = -2{\cdot}80584\ 61603\ 9 + {\cdot}01425\ 58610\ 7x^2 + {\cdot}00000\ 00662\ 60x^3 - {\cdot}00000\ 00516\ 08x^4,$$

whence we obtain

$$x = -2{\cdot}80584\ 60209\ 7 + {\cdot}01425\ 58603\ 6x^2 + {\cdot}00000\ 00662\ 60x^3 - {\cdot}00000\ 00516\ 08x^4.$$

From this equation x is easily obtained by the usual methods of successive approximation. The value we find is

$$x = \cos 2\sigma = -2{\cdot}70178\ 48031\ 8.$$

We thus see that σ is a complex quantity; its numerical value may be obtained from the usual tables. We find, in fact, that to four places of decimals,

$$\sigma = \frac{\pi}{2} + 0{\cdot}8255i, \quad \text{where } i = \sqrt{-1}.$$

Knowing now the numerical value of $\cos 2\sigma$ we may compute the corresponding values of $\sin 2\sigma$, $\sin 4\sigma$, etc., by means of the usual relations:—

$$\sin 2\sigma = \pm i\sqrt{\cos^2 2\sigma - 1}, \quad \cos^2 2\sigma \text{ being greater than unity.}$$

$$\sin 4\sigma = 2 \sin 2\sigma \cos 2\sigma, \text{ etc.}$$

The ambiguous sign leads to two values of μ numerically equal but of different sign, as may be expected from the nature of the general solution of differential equations of the type considered. We shall see that the positive value of $\sin 2\sigma$ leads to the negative value of μ and *vice versa*.

The values we find are:—

$$\begin{aligned}
\sin 2\sigma &= \quad 2{\cdot}50990\ 86064\ 0\, i \\
\sin 4\sigma &= -\ 13{\cdot}56247\, i \\
\sin 6\sigma &= \quad 70{\cdot}7758\, i \\
\sin 8\sigma &= -368{\cdot}8\, i.
\end{aligned}$$

The expression which we have obtained for μ in terms of σ becomes, when for the Θ's their corresponding numerical values are inserted, the following:—

$$\begin{aligned}
\mu = &-{\cdot}02852\ 03942\ 5 \sin 2\sigma - {\cdot}00000\ 00212\ 52 \sin 4\sigma \\
&+{\cdot}00000\ 00003\ 162 \sin 6\sigma - {\cdot}00000\ 00000\ 00026 \sin 8\sigma \\
&= -{\cdot}07158\ 35829\ 9\, i \\
&\quad +{\cdot}00000\ 02882\ 3\, i \\
&\quad +{\cdot}00000\ 00223\ 8\, i \\
&= -{\cdot}07158\ 32723\ 8\, i.
\end{aligned}$$

Hence, to ten places of decimals (no consideration being taken of the sign),

$$\mu = {\cdot}07158\ 32724\, i.$$

In Hill's celebrated paper, already referred to, the corresponding constant c is given to fifteen places of decimals. To ten places of decimals its value is

$$c = 1{\cdot}07158\ 32774.$$

The discrepancy in the ninth place of decimals is probably due to our having neglected high order terms whose influence on the final result is appreciable. The connection between Hill's value of c and the value we have obtained for μ is easily seen. In Hill's case c is real, but in the present working μ was found to be a pure imaginary, which arises in the fact that Hill so changed his independent variable that the imaginary unit i was absorbed into it. It may also be noticed that the value we have obtained for μ is $\cdot 07158 \ldots i$ and not $1\cdot 07158 \ldots i$, as might be expected by comparison with Hill's value of c. But if we write the factor $e^{\mu z}$ in the form $e^{\lambda i z}$ we see that its secular nature is unchanged by the addition of unity, thus $e^{(\lambda+1)iz}$, and that in this respect the non-appearance of the unity is a matter of no consequence. The general solution is now $e^{(\lambda-1)iz}e^{-iz}u(z)$, of which the factor $e^{-iz}u(z)$ has the period π, as is demanded by the general theory, whereas $u(z)$ itself has only the period 2π. This indicates that the other factor, which is in general secular as regards the period π, should be taken in the form $e^{(\lambda+1)iz}$ and not as it originally arose, and that therefore an initial unity should be supplied to our value of μ. The fact that μ is imaginary shows that the solution is stable, so that its corrected value gives us a measure of the period of small oscillations about the truly periodic orbit.

The method of dealing with Hill's equation indicated above seems to be quite satisfactory, and can be used to any desired degree of precision once the more advanced terms in the series for μ and Θ_0 have been obtained to a sufficiently high order. In the present paper they have only been taken to the tenth order, but to obtain higher terms is, though laborious, a matter of no great difficulty. The expansions for Θ_0 and μ which we have obtained are suitable only for the particular cases, such as that dealt with by Hill, in which Θ_0 is nearly unity. It is, however, equally possible to obtain similar expansions which are suited to the cases in which the quantity Θ_0 approaches the values 4, 9, 16, etc.

The Mathematical Laboratory,
University of Edinburgh:
1915 *February* 23.

Brilliant Meteors observed in 1914. By Fiammetta Wilson.

(*Communicated by Mr. W. F. Denning.*)

The following are the details of very brilliant meteors recorded at Bexley Heath, and a few during a voyage to Las Palmas, during the past year. It seems desirable to record such observations, so that comparisons may be made with similar materials obtained at other stations, and calculations made as to the real paths in cases where duplicate records can be found. Every effort was made to obtain accuracy in noting the apparent positions of the flights. The durations in most cases were made by means of a stop-watch, and in the case of the slow meteors it is believed that the times of flight as given are within small limits of error. The swifter meteors are so extremely brief in their visible flights that to attempt to register their duration by any very exact instrumental method seems futile, and I have merely given the roughly estimated durations.

In a number of instances the radiant points are given from comparisons with other observations of the same objects, which enabled the real paths to be computed by Mr. Denning, and these I have marked with an asterisk. In other cases the positions given are simply the probable or possible places derived from the durations of flight and aspect of the meteors. Some of these radiants may not be correctly attributed, and it is to be hoped that further observations of the same objects may be forthcoming, so that the radiants selected may be corroborated or otherwise.

During the year 1914 I observed 1060 meteors in 450 hours of watching, 81 accordances. The hourly rate may be considered very small, but this is easily accounted for by the fact that observations were very often maintained in spite of strong moonlight and a nearly all-clouded sky. So long as a few stars showed themselves, I considered the prospect sufficiently inviting to watch, and it is remarkable that on many such occasions I was rewarded by the view of one or two *specially* brilliant meteors. Out of the 1060 meteors observed, 103 were all above first magnitude, very many as bright or brighter than ♃, and several much brighter than ♀; this is a far higher percentage of fireballs than during any other year I have observed.

Probably I would have been enabled to record several other brilliant meteors during the latter end of the year, but for the fact that since the war my husband has been unable to help me—consequently, whilst I have been writing down notes recording one meteor, another may have eluded me.

No.	Date. 1914.	G.M.T.	Mag.	Appearance. R.A.	Appearance. Dec.	Disappearance. R.A.	Disappearance. Dec.	Length of Path.	Duration.	Colour.	Probable Radiant. R.A.	Probable Radiant. Dec.	Remarks.
		h m		°	°	°	°	°	s		°	°	
1	Jan. 3	17 33	>1	$237\frac{1}{2}$	+ 3	$241\frac{1}{4}$	± 0	$5\frac{1}{2}$	0·5	Yellow.	221	+14	
2	4	8 34	♂	76	−15	58	−28	21	0·8	,,	133	+30	
3	17	7 2 30	>1	$67\frac{1}{4}$	− 7	$68\frac{1}{4}$	$-14\frac{1}{2}$	$7\frac{1}{2}$	0·8	,,	29	+70	
4	17	10 31	See remarks.	$51\frac{1}{4}$	$-5\frac{1}{2}$	$52\frac{3}{4}$	$-17\frac{3}{4}$	$12\frac{1}{2}$	3	Blue head, yellow train.	29	+70	Appeared first = 1m, then = ♀.
5*	Feb. 5	7 59	♀	$73\frac{3}{4}$	+12	$38\frac{1}{2}$	± 0	36	1·5	Yellow.	174	+20	
6	22	8 10	♂	$158\frac{1}{2}$	$+32\frac{1}{2}$	187	+29	25	2·5	Bright red yellow.	110	+22 or Orion.	
7	26	11 40	♄	Approx. 144	−20	Approx. 145	−26	$6\frac{1}{2}$	1	Red yellow.	Cancer or Lynx.		Very low on the horizon, and slow movement. Too low in horizon mist to be anything but approximate.
8	27	10 35	>1	355	$+55\frac{1}{4}$	343	$+44\frac{1}{2}$	12	1	Red yellow.	106	+52	
9	Mar. 14	11 18	See remarks.	305	$+76\frac{3}{4}$	350	$+55\frac{3}{4}$	27	3	Green blue opalescent.	206	+55	Began = 3m, then brightened to 2× ♀.
10	14	11 34	>1	129	$+45\frac{1}{2}$	125	$+46\frac{1}{4}$	$2\frac{1}{2}$	1	Yellow.	Leo Minor or Virgo.		Part of course lost in cloud, obviously a long-pathed meteor.
11	18	9 36	−1·6	$231\frac{1}{4}$	+28	$247\frac{3}{4}$	$+42\frac{3}{4}$	20	1	,,	205	− 8	
12	28	9 38	>1	72	+50	69	+47	$5\frac{1}{2}$	0·5	...	Ursa Major.		Brilliant flash only.
13	April 4	10 46 30	>1	$211\frac{1}{2}$	+45	215	$+1\frac{3}{4}$	43	1	Yellow.	Perseus.		Halting course, seen through cloud.
14†	20	11 6	>1	$192\frac{1}{2}$	+ 4	$177\frac{1}{2}$	$-6\frac{1}{2}$	17	1	Red yellow.	270	+31	Long streak.
15†	20	11 47	♀	$291\frac{1}{2}$	$+27\frac{3}{4}$	290	+14	14	5	Greenish blue.	105	+53	

16†		22	9 12	♀	222½	+30	154½	+83	57	4	,,	227	−30	
17†		29	10 46	> δ	195	−43½	180	−49½	11	2	Red yellow.	218	−31	
18	May	18	9 26	♀	170	−25	171¼	−28	3½	2	Yellow.	Leo.		Flashed out 3 times, each time as bright as ♀.
19*		18	10 38	> Sirius.	300	+41	326	+56½	22	3	Red yellow.	271	− 2	
20*	June	3	10 46	See remarks.	33	+59½	80	+46	29½	6·4	Yellow.	281	−25	Began = δ, brightened = ♀, faded to = Sirius.
21		4	11 7	♀	103	+32	105	+20	12	0·5	,,	β Cephei or Cygnus.		Seen through cloud.
22		20	10 33	> ♀	200	− 6	208¾	−15	11½	1	Blue.	Leo Minor or Lynx.		Ended in cloud, no train.
23*		25	10 51 30	> 1	175	+21	170	+21½	5	0·5	,,	260	−24	
24*		25	11 27 30	♀	353	+54¼	35	+72	24	1·5	,,	342	+39	Streak.
25*		25	11 53 30	♃	210	+65	205	+46	20	1	Yellow.	354	+77	
26*		25	11 57 30	♃	221	+19½	207½	+19	13	1	,,	350	− 8	,,
27		27	11 50	> 1	267½	−11	275	−14	8	2	,,	α Serpentis or 221	+13	
28*	July	17	11 10	> 1	28	+71½	69	+68¾	17	0·5	White.	316	+30	
29		18	10 4	Sirius.	7	+34	25	+40	16	1·5	Yellow.	305	−12	
30*		22	10 35	♃	252	− 2	240	−11½	14	0·5	Yellow head, very white streak.	332	+31	,,
31		25	10 20	> ♃	283¾	+ 5½	274	+ 3¼	10	1	Yellow.	322	+10	
32		28	10 56	♃	305	+38	285	+29	18	0·8	...	Triangulid or Arietid.		In cloud.

* Meteors doubly observed are marked thus * in col. 1.

† Nos. 14–17 seen at sea in southern latitudes.

No.	Date. 1914.	G.M.T.	Mag.	Appearance. R.A.	Appearance. Dec.	Disappearance. R.A.	Disappearance. Dec.	Length of Path.	Duration.	Colour.	Probable Radiant. R.A.	Probable Radiant. Dec.	Remarks.
		h m		°	°	°	°	°	s		°	°	
33*	July 29	10 46	♀	172	+55½	160	+42	15	1·5	Yellow.	326	−12	
34	Aug. 2	11 28	δ	24	+51¾	23	+50¾	1½	0·5	White.	32	+56	Brilliant streak, showing a deep white furrow in the sky. Short apparent path.
35	2	11 51	δ	4	+ 0½	7½	+10	10	0·8	Red yellow.	358	21	Impeded movement, rather undulating.
36	9	9 46	♀	257⅓	−18½	...	...	...	Stationary.	Yellow.	257½	−18½	
37	11	9 27 30	♀	177½	+35	195	+10	29½	1	,,	β Perseid.		Head broke into fragments.
39*	11	10 20	♄	41	+66	140¾	+81½	23	1·3	,,	28	+36	Broke off in middle of course, then went on again.
40	11	10 59	Capella	58	+49¾	86	+52½	17	2	,,	Aquarius.		
41	11	11 6	Sirius.	256¼	+25	253¾	+12	13	0·5	...	Perseid.		Streak.
42*	11	11 25	>1	315	+34	302½	+14¾	21	0·5	White.	47	+61	Rapid streaked meteor.
43	11	11 26	Capella	295	+ 5	310	− 3¾	17½	0·5	...	Hercules.		Streak.
44	11	11 44 30	♀	198½	+57¼	202	+48½	9	0·5	...	Perseid.		
45*	11	11 45	>1	218¾	+40	221¼	+30	10½	0·4	White.	47	+56	,,
46*	12	8 50	♀	315	+20	301¼	− 3	27	0·8	...	48	+59	Streak endured 1·5 secs.
47	12	9 29	Vega.	273	+30	263	+10	20½	0·5	...	Perseid.		Streak.
48*	12	10 22	>Sirius.	225	+27¼	220½	+14	14	0·5		48	+58	,,
49*	12	10 33 30	♀	224	+62	227	+38	24	0·3	...	48	+57	Streak endured 2 secs.
50*	12	10 35 30	>1	173	+59	185¼	+44	15½	Very rapid indeed.	...	49	+57	Streak.

51*	12	10 52 30	>1	240¾	+15½	236¼	+10	6½	Very very rapid.	...	280	+43	
52	12	11 13	♆	12	+78	84	+59	29½	1·5	...	280	+43	
53	12	11 35	Vega 0·2	281¼	+38½	270	+23	17	0·5	...	Perseid.		,,
54	13	9 6	Sirius.	346½	+63¾	360	+56½	10	0·5	...	Draco.		
55	13	9 38	>1	168	+46½	213½	+36	35	0·5	...	140	+42	,,
56*	13	9 52	♆	167½	+36	168¾	+30	6	0·4	...	44	+39	A brilliant flash.
57	13	10 10 30	Vega.	13	+67	27	+63	7	1	Blue.	334	+70	
58	13	10 18	♆	336¼	+23	330	+ 7½	16	0·4	...	Perseid.		Streak endured 0·5 sec.
59	14	8 59	♆	5	+78½	280	+71½	20	2	...	,,		Streak.
60	14	9 23	♆	265	−13	260½	−20	9	0·8	...	,,		
61	14	9 25	♆	93	+52	87	+49	6	0·5	...	120	+60	Seen through mist obscuring stars.
62*	14	9 34	>♀	6	+33½	356	+20	15	Fairly rapid.	..	44	+59	Streak endured 20 secs.
63*	14	9 50	3>♀	330	+ 3	337¾	+ 1	8	2·5	Red yellow.	296	+10	
64	16	10 48	♆	127	+57	138	+51¼	9	0·5	...	Perseid.		Streak.
65	17	9 27	Vega.	300	−15	311¼	−25	15½	0·5	...	Hercules (?)		,,
66*	17	9 38	♀	285½	+11	286½	− 3	14	Very swift.	...	282	+44	Streak endured 3 secs.
67*	17	10 34	Sirius.	300¼	−15	308	−20	9	0·5	...	281	+46	
68	17	10 36	Vega.	291¾	+19	278¾	+ 7	17	0·5	...	26	+43	Streak endured 1 sec.
69	17	10 50	♀	248¾	+33	226¼	+23	23	1	...	280	+43	Streak, halting impeded motion of meteor.

* Meteors doubly observed are marked thus * in col. 1.

No.	Date. 1914.	G.M.T.	Mag.	Appearance. R.A.	Appearance. Dec.	Disappearance. R.A.	Disappearance. Dec.	Length of Path.	Duration.	Colour.	Probable Radiant. R.A.	Probable Radiant. Dec.	Remarks.
		h m		°	°	°	°	°	s		°	°	
70	Aug. 23	9 14	1	345½	+25	337½	+10	16	0·5	Yellow.	70	+66	Streak.
71	23	9 15	½>♀	342½	−14	Hidden by house.		...	Apparently slow.	Green blue.	...	...	Merely glimpsed this.
72	23	10 23	>1	72½	+45	75	+39	7	0·5	Red yellow.	280	+43	Streak.
73	24	9 37	1	330	−14	336	−21	9	1·5	Green.	280	+43	
74	28	10 23	1	325	−12	320	−16·30	6	1	Yellow.	26	+42	
75*	28	10 25	1	350	+47	285	− 2½	74	Rapid.	,,	76	+33	,,
76	Sept. 1	9 9	Sirius.	27	+29	359	+ 9	31	Very swift.	,,	73	+43	Streak endured 2 secs.
77*	8	8 55	♀	303¼	+ 3½	272½	−17	35	7	,,	348	+ 3	
78	15	9 6 30	>Sirius.	301¼	−31	Stationary.				Blue.	301¼	−31	
79	16	8 10	>Arcturus.	10	+47¾	63	+48¼	33	5	Yellow.	302 or 283	−10 −27	
80*	16	9 37	>1	347	+61¼	354	+77	16	1	,,	343	+22	
81	22	9 21	1	346¾	+43¼	338½	+63	21	Swift.	Blue green.	2	− 2	Streak endured 2·5 secs.
82	23	10 3	>1	244	+19¼	247	+10	10	0·5	Yellow.	Auriga.		
83*	24	7 55	1	236¼	+ 1	220	− 1	16½	1·5	,,	352	+ 2	
84	24	10 3 30	>1	212	+78	193½	+61½	17	0·5	Green.	Cepheus.		
85*	27	10 35	δ	310	−17	316¼	−25½	11	1	Yellow.	192	+56	
86	27	10 48	>1	52	+28	49¾	+21¾	7	0·5	,,	62	+50	Streak endured 0·8 sec.
87	Oct. 12	9 1 30	1	360	+ 9	14	+ 8	14	2·5	Green yellow.	ε Pegasid.		
88	18	8 22	<1	345	+78¼	290	+65½	19	0·8	,,	74	+30	Streak lasted 2 secs.

89		18	9	42	Vega.	351¾	− 3¼	355	−14¼	12	0·8	Yellow.	315	+62	
90		18	10	8	> ♅	From thick clouds in N.E.		308¾	− 7½	...	1	Blue.			
91*	Nov.	12	10	5	> Vega.	95	+62	139	+49	26	1	Green blue.	320	+41	
92		18	7	30	Capella.	290	− 8	310	−17	22	0·8	White.	Serpens or Böotes.		
93	Dec.	1	9		> 1	315	+56	318¾	+36	20	0·5	Yellow.	β Ursæ Majoris.		
94		3	8	7	> ♀	103	+28	109	+30	6	1	,,	70	+11	Broke into 2 parts.
95		5	6	47	♅	20	− 4	17	− 6	4	0·8	,,	32	+ 6	
96		5	9	33	> 1	120	+80	195	+70	19	0·4	,,	α–β Perseid.		A flash.
97		7	8	25	♅	45	+17	8	+ 7¾	38	2	Blue green.	90	+18	Streak.
98*		15	6	25	♄	270	+36½	240	+43¾	25	4·5	Red yellow.	336	−12	
99		15	7	4	> ♄	64	+10	52	± 0	14	3·5	...	91	+31	
100		19	10	3	Vega.	106	+70	133	+39½	34½	Slow and impeded.	...	322	+10	Stopped near end of course, then began again.
101		19	10	42	30 ♄	101	+45	111¾	+30	18	1·5	...	Cassiopeid.		Streak.
102		29	6	59	2 > ♅	94	+43	91½	+24½	19	2·5	Yellow.	261	+61	
103*		31	11	14	1½ > ♀	324	+67	350½	+55	16	2	Green yellow.	228	+54	

* Meteors doubly observed are marked thus * in col. 1.

Bexley Heath:
1915 *March* 9.

The following Report was received too late for insertion in the Annual Report of the Council.

Sydney Observatory.

The reorganisation of the Sydney Observatory and the transference to a new site have been still further postponed, and so once again the work has been merely of a local character.

The scheme of public time signals mentioned in the last report is now in operation. Every telephone subscriber may now at any time of the day obtain correct standard time. If he merely wishes to check his watch or clock and asks for "Time," he is placed in communication with an operator who reads off the correct time to the nearest half minute from a clock controlled by the Observatory Standard Clock. If he requires time correctly to a fraction of a second and asks for "Exact Time," he is switched on to a high frequency buzzer which transmits the actual beats of the Observatory clock.

The negotiations for the transmission of wireless time signals similar to those of Eiffel Tower have been temporarily discontinued owing to the war.

During the year longitude signals were exchanged between the Sydney and Melbourne Observatories and several camps established in South Australia by the Government Astronomer of that State. In this work wireless radiations were used for the first time in Australia.

Observations of comets, both photographic and micrometric, were taken when necessary.

At the University a continuous course of lectures on Astronomy, both theoretical and practical, was given. This marks the commencement of the inclusion of this subject in the regular graduate course.

The popular educational side of the Observatory work has been continuously encouraged. In addition to Monday afternoons, when the instruments are always open for inspection by any of the public, evening receptions have been held two or three times a week. For these latter, owing to lack of accommodation, each party is necessarily limited in number, and engagements must be made and booked in advance. Special mention may be made of that rapidly growing institution, the Workers' Educational Union, with the various branches of which a visit to the Observatory is becoming very popular.

In conclusion, it must be once more placed on record that permanent work of a real scientific utility, suitable for such an institution as this, is not possible under present circumstances. The year, however, has not been altogether wasted, and present indications for an immediate commencement of the much-needed reorganisation are decidedly hopeful.

Erratum.—On Plate 15 of the present volume it should have been stated that the illustration of the Nebula was a drawing by W. H. Wesley from the Isaac Roberts photograph.